

JEE ADVANCED (PAPER - 1) DATE: 25.05.2023

KEY SHEET

CHEMISTRY

MATHEMATICS

SOLUTIONS PHYSICS

- 1. $m = \frac{f}{f+u}$ $f + f + (-30)$ $f - 30$ h_i + *f* f h_0 + f + (-30) f $=\frac{+f}{+f+(-30)}=\frac{f}{f-}$ 2 30 2 $h + \frac{d-h}{h}$ *f h f* $+\frac{u \Rightarrow \frac{h}{\left(\frac{h}{2}\right)} = \frac{1}{f-30} \Rightarrow \frac{h}{h} = \frac{1}{f-30}$ *d h f h f* $\Rightarrow \frac{d+h}{h} = \frac{f}{f-1}$ 1 30 *d f* \Rightarrow $\frac{ }{h} = \frac{ }{f - 30}$ \Rightarrow 3 = $\frac{f-f+30}{g}$ 30 $f - f$ *f* $=\frac{f-f+30}{f-30}$ [: slope of d – h graph from figure is 3] \Rightarrow 3*f* $-90 = 30$ \Rightarrow *f* $=$ 40*cm*
- 2. If a point charge is at $|a, 2a,$ 2 *a a a* $\left(a,2a,\frac{a}{2}\right)$ then given surface is $1/8th$ of a square surface of side a

3. Velocity of point 'A'
$$
V_A = \sqrt{V^2 + \omega^2 R^2} = V\sqrt{2}
$$

Normal acceleration of point A,

$$
a_A(n) = \omega^2 R \cos 45^\circ + \alpha R \cos 45^\circ - a \cos 45^\circ
$$

$$
a_B = \omega^2 R = V^2
$$

$$
a_{_{A(n)}} = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}R}
$$

radius of curvature of trajectory of point 'A' relative to the ground is

$$
r = \frac{(V_A)^2}{a_{A(n)}} = \frac{(V\sqrt{2})^2}{\frac{V^2}{\sqrt{2}R}} = 2\sqrt{2}R
$$

4.
$$
K.E_{\alpha} = \frac{(Q - \Delta E)Y}{Y + 4}
$$

$$
= \frac{(8 - 2.4)220}{224} = 5.5
$$

5 & 6.

At the highest point velocity of the ball is $u\cos\theta = 5\sqrt{3m}/s$ Since POQ is an equilateral triangle

Range of the ball is 2 sin 2 2 $\frac{L}{2}$ = $PQ = \frac{u}{2}$ *g* $= PO = \frac{u^2 \sin 2\theta}{\sin 2\theta}$

$$
=10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3}
$$

$$
\Rightarrow L = 10\sqrt{3}
$$

7 & 8. There is a path difference between rays before incidence on the slit and it is equal to

$$
\Delta x_1 = d \sin 30^\circ = \frac{d}{2} = \frac{10^{-4}}{2} = 50 \times 10^{-6} m = 50 \,\mu m
$$

Also path difference introduced by upper $slab = 20.4(1.5-1) = 10.2 \mu m$

Let thickness of lower slab is 't', then path difference introduced = $t(\mu-1) = \frac{t}{2}$ $t(\mu-1) = \frac{t}{2}$

It is given that central maxima is above 'O'

 $_1 + 10.2$ 2 $\therefore \Delta x_1 + 10.2 - \frac{t}{2} =$ Path difference at 'O'

By Intensity relation $3I = 4I + I + 2 \cdot \sqrt{4I \cdot I} \cos \theta$, $\theta =$ phase difference

$$
\cos \theta = -\frac{1}{2}
$$

For maximum thickness for lower slab

$$
\therefore \theta = \frac{2\pi}{3}
$$

$$
\therefore \frac{2\pi}{\lambda} \cdot \Delta x = \frac{2\pi}{3}
$$

\n
$$
\Delta x = \frac{\lambda}{3} = 0.2 \ \mu m \Rightarrow 0.2 \ \mu m = 60.2 \ \mu m - \frac{t}{2}
$$

\n
$$
\therefore t = 120 \ \mu m
$$

\n
$$
I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \theta
$$

\n
$$
\therefore I_{\text{max}} = 4I + I + 4I = 9I
$$

\n
$$
I_{\text{min}} = 4I + I - 4I = I
$$

9 & 10. Using conservation of mechanical energy

$$
mg\frac{L}{2} = \frac{1}{2}\frac{ML^{2}}{3}\omega_{0}^{2}
$$
\n
$$
\Rightarrow \qquad \omega_{0} = \sqrt{\frac{3g}{L}}, \ \omega = \sqrt{\frac{3g}{2L}} = \sqrt{\frac{3g}{2\sqrt{2r}}}
$$
\n
$$
L = \sqrt{2}r
$$

11. Acceleration due to gravity at any point is the magnitude of gravitaitonal intensity at that point.

$$
\left(\frac{1}{r}\right)
$$

\n
$$
dm = 4\pi r^2 dx \frac{\sigma_0}{r}
$$

\n
$$
= 4\pi \sigma_0 r dr
$$

\n
$$
m = 4\pi \sigma_0 \left(\frac{r^2}{2}\right)^r
$$

\n
$$
m = 2\pi \sigma_0 \left(x^2 - R^2\right)
$$

 $=\frac{3(L-r)^{2}(\omega_{0}+\omega)}{2}$ $\omega_{0} - \omega$ 2 (ω_{0}

 2 (ω_{0} M $3(L-r)^2(\omega_0 + \omega)$ 3 m $L^2(\omega_0 - \omega)$ 2

$$
\Rightarrow E\left(4\pi r^2\right) = 4\pi G\left(M+m\right)
$$

\n
$$
E\left(r^2\right) = G\left[M + 2\pi\sigma_0 r^2 - 2\pi\sigma_0 R^2\right]
$$

\n
$$
E = \frac{G}{r^2}\left[M + 2\pi\sigma_0 r^2 - 2\pi\sigma_0 R^2\right]
$$

\n
$$
E = G\left[\frac{M}{r^2} + 2\pi\sigma_0 - \frac{2\pi\sigma_0 R^2}{r^2}\right]
$$

\n
$$
E = G\left[2\pi\sigma_0 + \frac{1}{r^2}\left(M - 2\pi\sigma_0 R^2\right)\right]
$$

\n
$$
\Rightarrow \text{if } \sigma_0 = \frac{M}{2\pi R^2}, \text{ E is independent of r}
$$

12. For smaller sphere, before (covering $P_{point\,Source} = P_{radiated}$) $\therefore P = \sigma \left(4 \pi R^2 \right) T_0^4$ After covering, for outer shell : $P_{point\,Source} = P_{radiated}$ $\therefore \sigma \bigl(4\pi R^2 \bigr) T_0^4 = \sigma \Bigl(4\pi \bigl(2R \bigr)^2 \Bigr) T^4$ 4 $A T^{4}$ \rightarrow T 4 0 $n_0^4 = 4$ *T* $T^* = 4T^* \Rightarrow T =$

2

After covering, temperature of inner sphere will be greater than T_0

13. Current through the inductor 'L₀' in the steady state after closing the switch-2 will be I_s *R* $=\frac{\varepsilon}{\sqrt{2}}$

Now,
$$
2L \frac{di_2}{dt} = L \frac{di_1}{dt} + i_1 R_0
$$

\n $2L \int_0^{\varepsilon/R} di_2 - L \int_{\varepsilon/2R}^0 di_1 = R_0 \int i_1 dt$
\n $2L \frac{\varepsilon}{R} - L \left(0 - \frac{\varepsilon}{2R} \right) = R_0 \Delta q$
\n $\frac{5\varepsilon L}{2R} = R_0 \Delta q \Rightarrow \Delta q = \frac{5\varepsilon L}{2RR_0}$
\n $\Delta q = \frac{5\varepsilon L}{2R^2}$

14. Immediately after the first collision between upper bar and the wall, direction of motion of upper bar is reversed and both the bars are moving with their initial speeds in opposite directions. Now slipping starts. They have equals speed in opposite direction before the lower bar collides with wall. Work done by friction on both the bars is equal to their change in K.E.

Final speed of the bars is zero if $u \leq \sqrt{2\mu g}$ as both the bars come to rest before the lower bar collides with the wall.

$$
If u \ge \sqrt{2\mu gl_0},
$$

$$
2 \times \frac{1}{2} m(v^2 - u^2) = -2 \left(\mu mgl\right)
$$

$$
v = \sqrt{u^2 - 2\mu gl}
$$

After the lower bar collides with the wall. They have equal speed in the direction away from the wall.

15.
$$
\frac{\lambda}{4} = 0.1 \Rightarrow \lambda = 0.4m
$$

From the graph time period is $T = 0.2s$ and amplitude of stationary wave is $2A = 4cm$

Equation of the standing wave is $y(x,t) = -2A\cos\left(\frac{2\pi}{x},x\right)\sin\left(\frac{2\pi}{x}\right)$ 0.4 0.2 $y(x,t) = -2A\cos\left(\frac{2\pi}{0.4}x\right)\sin\left(\frac{2\pi}{0.2}t\right)$

Transverse velocity of the particle is given by *y* $V = \frac{oy}{g}$ *t* $=\frac{\partial}{\partial}$

using point No-4

17.

(3) wrong as no adiabatic process is involved

$$
E = 5\left(\frac{1}{2}mv^2\right) + \frac{1}{2}kx^2
$$

\n
$$
T = 2\pi\sqrt{\frac{5m}{k}}
$$

\n18. When the switch S is open
\nEnergy $U_1 = U_0 + \frac{kQ^2}{2R} + \frac{kQ^2}{4R} + \frac{kQ^2}{R} - \frac{kQ^2}{2R} - \frac{kQ^2}{2R}$
\n
$$
= U_0 + \frac{3kQ^2}{4R}
$$

\nEnergy $U_2 = U_0 + \frac{kQ^2}{2R} + \frac{kQ^2}{4R} - \frac{kQ^2}{R} + \frac{kQ^2}{2R} - \frac{kQ^2}{2R}$
\n
$$
= U_0 - \frac{kQ^2}{4R}
$$

\nLoss of energy = $U_1 - U_2 = \frac{kQ^2}{R}$
\n
$$
= \frac{9 \times 10^9 \times 400 \times 10^{-12}}{9 \times 10^{-2}} = 40J
$$

\n19. $E = \sigma T^4$
\n
$$
\Rightarrow \frac{E_2}{E_1} = \frac{T_2^4}{T_1^4} = 256 = 4^4
$$

\n
$$
\Rightarrow T_2 = 4T
$$

\nAlso, $\lambda_1 T_1 = \lambda_2 T_2$ gives $\lambda_2 = \frac{\lambda_1}{4} = 1000A^0$
\n
$$
KE_{max} = \frac{hc}{\lambda_2} - \phi
$$

\n
$$
\Rightarrow \frac{12400}{1000} - \phi = 12.4 - \phi - - - (1)
$$

\n
$$
\Delta E_{2\to 4} = 13.6 \times \left[\frac{1}{2^2} - \frac{1}{4^2}\right] = 13.6 \times 4 \times \frac{3}{16} = 10.2eV - - - (2)
$$

\nFrom (1) and (2) 12.4 - $\phi = 10.2$
\n
$$
\Rightarrow \phi = 2.2eV
$$

CHEMISTRY

20. Conceptual
\n21.
$$
A(OH)_2 \square A^{2+} + 2OH^{-}
$$
; $K_{sp} = 4 \times 10^{-12}$
\n $B(OH)_3 \square B^{3+} + 3OH^{-}$; $K_{sp} = 1 \times 10^{-33}$
\n $\therefore (Ksp)_{A(OH)_2} >> (Ksp)_{B(OH)_3}$
\n $\therefore y << x$
\n22. Stable free radical
\n23. Anti group migration
\n24 & 25. 2NaN₃ \rightarrow 2Na + 3N₂
\n2NaNO3 + 10 Na \rightarrow 6Na₂O + N₂
\n26. 6AgI
\n27. 2-Chloro-buta-1,3-diene
\n28. Let the reaction to be of 1st order
\n $K = \frac{2.303}{10} \log \frac{25.6}{16} = 0.047$
\n $K = \frac{2.303}{20} \log \frac{25.6}{10} = 0.047$
\n29. $t_{1/2} = \frac{0.693}{0.047} = 14.745$ min
\n30. (A) Cl₂O₆ contain ClO₂⁺ ClO₄, ClO₂^{*} is planar
\n(B) Due to the back bonding N(SiH₃)s weak base
\n(C) Both HNO₂ and HClO₃ disproportional on heating
\n(D) In both NO₂ and ClO₂ central atoms are in sp² hybridisation
\n31. $\frac{CrO_4^{2-}$ or Cr₂O₂⁻² + H₂O₂ + H⁺ \longrightarrow CrO₅ + H₂O
\n $\frac{CrO_5 + H^+ \longrightarrow \frac{Cr^3}{(\text{Gness solar})^2} + O_2(g) + H_2O$
\n34. Nitrobenzene cannot give Friedel-Crafts reaction
\n35. Blane rule
\n36. in (b) N₂

Si forms CCP and C occupies alternate THV

37.

40.
$$
P(x) = -x^3 + 5x + 5
$$

$$
\frac{1}{2}
$$

 $NH₂⁺$

41.
$$
\cos^2 \beta - \cos^2 \alpha = \sin(\alpha + \beta)\sin(\alpha - \beta)
$$
, $a_n = \frac{1}{\sin(\frac{\pi}{2n})}\left[\frac{1}{\cos^2(\frac{(n-1)\pi}{2n})}-1\right]$

$$
a_n = \left(\frac{1}{\sin^3 \frac{\pi}{2n}} - \frac{1}{\sin\left(\frac{\pi}{2n}\right)}\right)
$$

 $\sqrt{2}$

45.

The equations of the bisectors are given by $x - y = 0$ and $x + y + 2 = 0$.

These bisectors intersect at the point $P(-1, -1)$

Focus S, is the foot of ⊥ from P to MN i.e. point of intersection of lines MN $(3x - y - 2 = 0)$ and PS $(x + 3y + 4 = 0)$.

$$
\therefore \qquad \text{Focus } S \equiv \left(\frac{1}{5}, -\frac{7}{5}\right)
$$

Circle passing through PMN is $(x-0)(x-1)+(y-1)(y+2)=0$

or
$$
x^2 + y^2 - x + y - 2 = 0
$$

Hence, equation of tangent at $(-1, -1)$, which is also the equation of directrix is $3x + y + 4 = 0$.

47.

Sol.
$$
(x-2) = \sqrt{(x-7)^2 + (y+2)^2}
$$

It is parabola
1
1
2
2
2
2
2
2
2
2
Q
2
Q
2
Q
2
Q
2
Q
2
Q
Q
2
Q
Q
Q
Q
Q
Q
Q
Q
Q
Q

48.
$$
\arg\left(\frac{z_1 - (7 - 2i)}{z_2 - (7 - 2i)}\right) = \pi
$$

 $(1,0)$ lies on hyperbola 49.

> Equation of tangent to hyperbola at $(1,0)$ is $x+y-1=0$ Homogenize the circle $x^2 + y^2 + 2gx + 2fy = 0$ with $x + y - 1 = 0$ and compare that with given pair of asymptotes

50.
$$
(3+4)\left(\frac{(\sin^{-1}x)^2}{3} + \frac{(\cos^{-1}x)^2}{4}\right) \ge \frac{25\pi^2}{4}
$$

\n
$$
\Rightarrow \sin^{-1}x = \frac{15\pi}{14}, \cos^{-1}x = \frac{20\pi}{14}
$$

\n51.
$$
a_n^2 - b_n^2 = \text{Re}(2+i)^{2n}, a_nb_n = \frac{1}{2}\text{Im}((2+i)^{2n})
$$

\n52.
$$
\max\{f(x), g(x)\} = \frac{f(x) + g(x) + |f(x) - g(x)|}{2}
$$

\n
$$
f(x) = 2 \max\{|x - 1| + |x - 3|, |2x - 1|\}
$$

\n53.
$$
I_1 = \int_0^1 \cos(\pi - \pi \cos^2 x) dx = -I_1
$$

\n
$$
I_2 = \int_0^1 \cos(\pi (2\cos^2 x)) dx, \quad I_2 = -\int_0^1 \cos(\pi \cos 2x) dx = -I_3
$$

54.
$$
f_n = f_{n-1} + f_{n-2}, \quad f_1 = 2, f_2 = 3
$$