

**JEE ADVANCED (PAPER - 1) DATE: 25.05.2023** 

## KEY SHEET

## **PHYSICS**

1	В	2	C	3	В	4	C	5	5
6	10	7	120	8	9	9	1.5	10	3
11	AD	12	AC	13	AC	14	ABCD	15	ACD
16	AB	17	2	18	4	19	1		

### **CHEMISTRY**

20	В	21	В	22	В	23	С	24	67.2
25	34	26	9	27	2	28	1	29	14.70 TO 14.80
30	ACD	31	ACD	32	вс	33	ABCD	34	CD
35	BCD	36	3	37	6	38	1		

### **MATHEMATICS**

39	A	40	В	41	В	42	C	43	99
44	19	45	1.20	46	4	47	25	48	3.14
49	AC	50	AD	51	AD	52	ABC	53	ABC
54	CD	55	2	56	<b>1</b> C	57	2		

# SOLUTIONS PHYSICS

1. 
$$m = \frac{f}{f+u}$$

$$\frac{h_i}{h_0} = \frac{+f}{+f+(-30)} = \frac{f}{f-30}$$

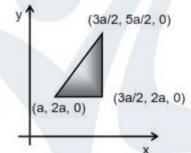
$$\Rightarrow \frac{h + \frac{d-h}{2}}{\left(\frac{h}{2}\right)} = \frac{f}{f-30} \Rightarrow \frac{d+h}{h} = \frac{f}{f-30}$$

$$\Rightarrow \frac{d}{h} = \frac{f}{f-30} - 1$$

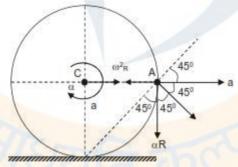
$$\Rightarrow 3 = \frac{f-f+30}{f-30} \quad [\because \text{ slope of d-h graph from figure is 3}]$$

$$\Rightarrow 3f - 90 = 30 \quad \Rightarrow f = 40cm$$

2. If a point charge is at  $\left(a, 2a, \frac{a}{2}\right)$  then given surface is  $1/8^{\text{th}}$  of a square surface of side a



3. Velocity of point 'A'  $V_A = \sqrt{V^2 + \omega^2 R^2} = V\sqrt{2}$ 



Normal acceleration of point A,

$$a_A(n) = \omega^2 R \cos 45^0 + \alpha R \cos 45^0 - a \cos 45^0$$

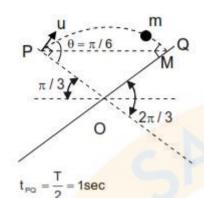
$$a_{A(n)} = \frac{\omega^2 R}{\sqrt{2}} = \frac{V^2}{\sqrt{2}R}$$

: radius of curvature of trajectory of point 'A' relative to the ground is

$$r = \frac{(V_A)^2}{a_{A(n)}} = \frac{(V\sqrt{2})^2}{\frac{V^2}{\sqrt{2}R}} = 2\sqrt{2}R$$

4. 
$$K.E_{\alpha} = \frac{(Q - \Delta E)Y}{Y + 4}$$
$$= \frac{(8 - 2.4)220}{224} = 5.5$$

5 & 6.



$$t_{PQ} = \frac{2u\sin\theta}{g}$$

$$\frac{2u\left(\frac{1}{2}\right)}{10} = 1$$

$$u = 10ms^{-1}$$

At the highest point velocity of the ball is  $u \cos \theta = 5\sqrt{3}m/s$ Since POQ is an equilateral triangle

Range of the ball is  $\frac{L}{2} = PQ = \frac{u^2 \sin 2\theta}{g}$ 

$$= 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3}$$
$$\Rightarrow L = 10\sqrt{3}$$

7 & 8. There is a path difference between rays before incidence on the slit and it is equal to

$$\Delta x_1 = d \sin 30^\circ = \frac{d}{2} = \frac{10^{-4}}{2} = 50 \times 10^{-6} m = 50 \,\mu m$$

Also path difference introduced by upper slab =  $20.4(1.5-1) = 10.2 \mu m$ 

Let thickness of lower slab is 't', then path difference introduced =  $t(\mu - 1) = \frac{t}{2}$ 

It is given that central maxima is above 'O'

∴
$$\Delta x_1 + 10.2 - \frac{t}{2}$$
 = Path difference at 'O'

By Intensity relation  $3I = 4I + I + 2 \cdot \sqrt{4I \cdot I} \cos \theta$ ,  $\theta = \text{phase difference}$ 

$$\cos\theta = -\frac{1}{2}$$

For maximum thickness for lower slab

$$\therefore \theta = \frac{2\pi}{3}$$

$$\therefore \frac{2\pi}{\lambda} \cdot \Delta x = \frac{2\pi}{3}$$

$$\Delta x = \frac{\lambda}{3} = 0.2 \,\mu m \Rightarrow 0.2 \,\mu m = 60.2 \,\mu m - \frac{t}{2}$$

$$\therefore t = 120 \,\mu m$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \theta$$

$$\therefore I_{\text{max}} = 4I + I + 4I = 9I$$

$$I_{\text{min}} = 4I + I - 4I = I$$

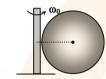
9 & 10. Using conservation of mechanical energy

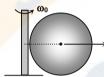
$$mg\frac{L}{2} = \frac{1}{2}\frac{ML^2}{3}\omega_0^2$$

 $\Rightarrow$ 

$$\omega_0 = \sqrt{\frac{3g}{L}} , \ \omega = \sqrt{\frac{3g}{2L}} = \sqrt{\frac{3g}{2\sqrt{2}r}}$$

 $L = \sqrt{2}r$ 





before collision

after collision

Using the definition of e

$$(L-r)\omega_0 = v - \omega(L-r)$$

$$v = (L - r) (\omega_0 + \omega) = (L - r)\omega(\sqrt{2} + 1) = r\omega = \sqrt{\frac{3gr}{2\sqrt{2}}}$$

$$r = \frac{6\sqrt{2}}{10} m$$

$$v = \sqrt{\frac{3 \times 10 \times 6 \times \sqrt{2}}{10}}$$

$$v = \sqrt{\frac{3 \times 10 \times 6 \times \sqrt{2}}{2\sqrt{2} \times 10}} = 3 \text{ m/s}$$

**Using COAM** 

$$I\omega_0 = I\omega + mv(L - r)$$

$$I(\omega_0 - \omega) = m(\omega_0 + \omega) (L - r)^2$$

$$\frac{ML^2}{3}(\omega_0 - \omega) = m(\omega_0 + \omega)(L - r)^2$$

$$\frac{M}{m} = \frac{3(L - r)^2(\omega_0 + \omega)}{L^2(\omega_0 - \omega)} = \frac{3}{2}$$

11. Acceleration due to gravity at any point is the magnitude of gravitaitonal intensity at that point.



$$dm = 4\pi r^2 dx \frac{\sigma_0}{r}$$

$$=4\pi\sigma_0 r dr$$

$$m = 4\pi\sigma_0 \left(\frac{r^2}{2}\right)^r$$

$$m = 2\pi\sigma_0 \left( x^2 - R^2 \right)$$

$$\Rightarrow E\left(4\pi r^{2}\right) = 4\pi G\left(M + m\right)$$

$$E\left(r^{2}\right) = G\left[M + 2\pi\sigma_{0}r^{2} - 2\pi\sigma_{0}R^{2}\right]$$

$$E = \frac{G}{r^{2}}\left[M + 2\pi\sigma_{0}r^{2} - 2\pi\sigma_{0}R^{2}\right]$$

$$E = G\left[\frac{M}{r^{2}} + 2\pi\sigma_{0} - \frac{2\pi\sigma_{0}R^{2}}{r^{2}}\right]$$

$$E = G\left[2\pi\sigma_{0} + \frac{1}{r^{2}}\left(M - 2\pi\sigma_{0}R^{2}\right)\right]$$

$$\Rightarrow \text{if } \sigma_{0} = \frac{M}{2\pi R^{2}}, \text{E is independent of } r$$

12. For smaller sphere, before (covering  $P_{Point Source} = P_{radiated}$ )

$$\therefore P = \sigma \left(4\pi R^2\right) T_0^4$$

After covering, for outer shell:  $P_{Point Source} = P_{radiated}$ 

$$\therefore \sigma \left(4\pi R^2\right) T_0^4 = \sigma \left(4\pi \left(2R\right)^2\right) T^4$$

$$\therefore T_0^4 = 4T^4 \Rightarrow T = \frac{T_0}{\sqrt{2}}$$

After covering, temperature of inner sphere will be greater than  $T_0$ 

13. Current through the inductor 'L<sub>0</sub>' in the steady state after closing the switch-2 will be  $I_s = \frac{\mathcal{E}}{R}$ 

Now, 
$$2L\frac{di_2}{dt} = L\frac{di_1}{dt} + i_1R_0$$

$$2L\int_0^{\varepsilon/R} di_2 - L\int_{\varepsilon/2R}^0 di_1 = R_0 \int i_1 dt$$

$$2L\frac{\varepsilon}{R} - L\left(0 - \frac{\varepsilon}{2R}\right) = R_0 \Delta q$$

$$\frac{5\varepsilon L}{2R} = R_0 \Delta q \Longrightarrow \Delta q = \frac{5\varepsilon L}{2RR_o}$$

$$\Delta q = \frac{5\varepsilon L}{2R^2}$$

14. Immediately after the first collision between upper bar and the wall, direction of motion of upper bar is reversed and both the bars are moving with their initial speeds in opposite directions. Now slipping starts. They have equals speed in opposite direction before the lower bar collides with wall. Work done by friction on both the bars is equal to their change in K.E.

Final speed of the bars is zero if  $u \le \sqrt{2\mu g l_0}$  as both the bars come to rest before the lower bar collides with the wall.

If 
$$u \ge \sqrt{2\mu g l_0}$$
,

$$2 \times \frac{1}{2} m \left( v^2 - u^2 \right) = -2 \left( \mu m g l \right)$$
$$v = \sqrt{u^2 - 2 \mu g l}$$

After the lower bar collides with the wall. They have equal speed in the direction away from the wall.

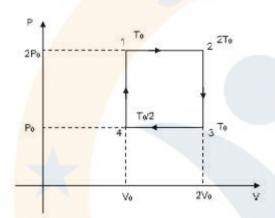
15. 
$$\frac{\lambda}{4} = 0.1 \Rightarrow \lambda = 0.4m$$

From the graph time period is T = 0.2s and amplitude of stationary wave is 2A = 4cm

Equation of the standing wave is 
$$y(x,t) = -2A\cos\left(\frac{2\pi}{0.4}x\right)\sin\left(\frac{2\pi}{0.2}t\right)$$

Transverse velocity of the particle is given by  $V_y = \frac{\partial y}{\partial t}$ 

16.



(4) 
$$\left| \frac{\Delta Q_{1 \to 2}}{\Delta Q_{3 \to 4}} \right| = \left| \frac{NC_P \Delta T_{1 \to 2}}{NC_P \Delta T_{3 \to 4}} \right| = \frac{T_0}{T_0 / 2} = 2$$

(2) 
$$\left| \frac{\Delta Q_{1 \to 2}}{\Delta Q_{2 \to 3}} \right| = \left| \frac{N C_p \Delta T_{1 \to 2}}{N C_v \Delta T_{2 \to 3}} \right| = \frac{C_p}{C_v} = \frac{5}{3}$$

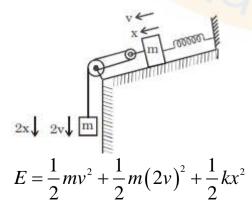
$$(1) W_{cycle} = P_0 V_o = nR \left[ \frac{T_0}{2} \right]$$

using point No-4

(3)

wrong as no adiabatic process is involved

17.



$$E = 5\left(\frac{1}{2}mv^2\right) + \frac{1}{2}kx^2$$
$$T = 2\pi\sqrt{\frac{5m}{k}}$$

18. When the switch S is open

Energy 
$$U_1 = U_0 + \frac{kQ^2}{2R} + \frac{kQ^2}{4R} + \frac{kQ^2}{R} - \frac{kQ^2}{2R} - \frac{kQ^2}{2R}$$

$$= U_0 + \frac{3kQ^2}{4R}$$

$$= U_0 + \frac{kQ^2}{4R} + \frac{kQ^2}{2R} + \frac{kQ^2}{4R} - \frac{kQ^2}{R} + \frac{kQ^2}{2R} - \frac{kQ^2}{2R}$$

$$= U_0 - \frac{kQ^2}{4R}$$

Loss of energy=
$$=U_1 - U_2 = \frac{kQ^2}{R}$$
  
=  $\frac{9 \times 10^9 \times 400 \times 10^{-12}}{9 \times 10^{-2}} = 40J$   
 $E = \sigma T^4$ 

19. 
$$E = \sigma T^{4}$$

$$\Rightarrow \frac{E_{2}}{E_{1}} = \frac{T_{2}^{4}}{T_{1}^{4}} = 256 = 4^{4}$$

$$\Rightarrow T_{2} = 4T$$

Also, 
$$\lambda_1 T_1 = \lambda_2 T_2$$
 gives  $\lambda_2 = \frac{\lambda_1}{4} = 1000A^0$ 

$$KE_{\text{max}} = \frac{hc}{\lambda_2} - \phi$$
  
 $\Rightarrow \frac{12400}{1000} - \phi = 12.4 - \phi - - - (1)$ 

$$\Delta E_{2\to 4} = 13.6 \times \left[ \frac{1}{2^2} - \frac{1}{4^2} \right] = 13.6 \times 4 \times \frac{3}{16} = 10.2eV - ---(2)$$

From (1) and (2) 
$$12.4 - \phi = 10.2$$

$$\Rightarrow \phi = 2.2eV$$

#### **CHEMISTRY**

21. 
$$A(OH)_2 \square A_x^{2+} + 2OH^-; K_{sp} = 4 \times 10^{-12}$$
  
 $B(OH)_3 \square B_y^{3+} + 3OH^-; K_{sp} = 1 \times 10^{-33}$   
 $\therefore (Ksp) >>> (Ksp)$ 

- 22. Stable free radical
- 23. Anti group migration

24 & 25. 
$$2\text{NaN}_3 \rightarrow 2\text{Na} + 3\text{N}_2$$
  
 $2\text{NaNO3} + 10 \text{ Na} \rightarrow 6\text{Na}_2\text{O} + \text{N}_2$ 

- 26. 6AgI
- 27. 2-Chloro-buta-1,3-diene
- 28. Let the reaction to be of 1<sup>st</sup> order

$$K = \frac{2.303}{10} \log \frac{25.6}{16} = 0.047$$
$$K = \frac{2.303}{20} \log \frac{25.6}{10} = 0.047$$

29. 
$$t_{1/2} = \frac{0.693}{0.047} = 14.745 \,\text{min}$$

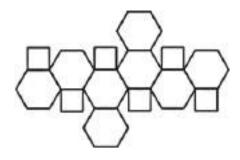
- 30. (A)  $Cl_2O_6$  contain  $ClO_2^+ClO_4^-$ ,  $ClO_2^+$  is planar
  - (B) Due to the back bonding N(SiH<sub>3</sub>)<sub>3</sub> weak base
  - (C) Both HNO2 and HClO3 disproportional on heating
  - (D) In both NO<sub>2</sub> and ClO<sub>2</sub> central atoms are in sp<sup>2</sup> hybridisation

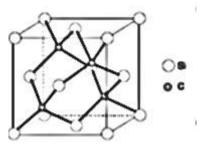
31. 
$$\underbrace{\operatorname{CrO}_{4}^{2-} \operatorname{or} \operatorname{Cr}_{2} \operatorname{O}_{7}^{2-}}_{(x)} + \operatorname{H}_{2} \operatorname{O}_{2} + \operatorname{H}^{+} \longrightarrow \operatorname{CrO}_{5} + \operatorname{H}_{2} \operatorname{O}$$

$$CrO_5 + H^+ \longrightarrow Cr^{3+} + O_2(g) + H_2O$$
(Green Soln Z)

- 34. Nitrobenzene cannot give Friedel-Crafts reaction
- 35. Blanc rule
- 36. in (b)  $N_2O$  is released
  - In (e) H<sub>2</sub> is released
  - In (f) NH3 is released

37.





Si forms CCP and C occupies alternate THV

38.

#### **MATHS**

40. 
$$P(x) = -x^3 + 5x + 5$$

41. 
$$\cos^2 \beta - \cos^2 \alpha = \sin(\alpha + \beta)\sin(\alpha - \beta), a_n = \frac{1}{\sin(\frac{\pi}{2n})} \left(\frac{1}{\cos^2(\frac{(n-1)\pi}{2n})} - 1\right)$$

$$a_n = \left(\frac{1}{\sin^3 \frac{\pi}{2n}} - \frac{1}{\sin \left(\frac{\pi}{2n}\right)}\right)$$

45. The equations of the bisectors are given by x - y = 0 and x + y + 2 = 0.

These bisectors intersect at the point P (-1, -1)

Focus S, is the foot of  $\perp$  from P to MN i.e. point of intersection of lines MN (3x-y-2=0) and PS (x+3y+4=0).

$$\therefore \quad \text{Focus S} \equiv \left(\frac{1}{5}, -\frac{7}{5}\right)$$

Circle passing through PMN is (x-0)(x-1)+(y-1)(y+2)=0

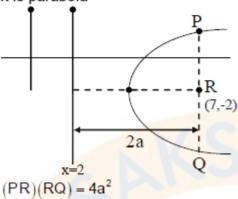
or 
$$x^2 + y^2 - x + y - 2 = 0$$

Hence, equation of tangent at (-1, -1), which is also the equation of directrix is 3x + y + 4 = 0.

47.

Sol. 
$$(x-2) = \sqrt{(x-7)^2 + (y+2)^2}$$

It is parabola



$$=4 \times \left(\frac{5}{2}\right)^2$$
$$=25$$

48. 
$$\arg\left(\frac{z_1 - (7 - 2i)}{z_2 - (7 - 2i)}\right) = \pi$$

49. (1,0) lies on hyperbola

Equation of tangent to hyperbola at (1,0) is x+y-1=0

Homogenize the circle  $x^2 + y^2 + 2gx + 2fy = 0$  with x + y - 1 = 0 and compare that with given pair of asymptotes

50. 
$$\left(3+4\right) \left(\frac{\left(\sin^{-1} x\right)^2}{3} + \frac{\left(\cos^{-1} x\right)^2}{4}\right) \ge \frac{25\pi^2}{4}$$

$$\Rightarrow \sin^{-1} x = \frac{15\pi}{14}, \cos^{-1} x = \frac{20\pi}{14}$$

51. 
$$a_n^2 - b_n^2 = \text{Re}(2+i)^{2n}, a_n b_n = \frac{1}{2} \text{Im}((2+i)^{2n})$$

52. 
$$\max\{f(x), g(x)\} = \frac{f(x) + g(x) + |f(x) - g(x)|}{2}$$
$$f(x) = 2\max\{|x - 1| + |x - 3|, |2x - 1|\}$$

53. 
$$I_{1} = \int_{0}^{1} \cos(\pi - \pi \cos^{2} x) dx = -I_{1}$$

$$I_{2} = \int_{0}^{1} \cos(\pi (2\cos^{2} x)) dx, \quad I_{2} = -\int_{0}^{1} \cos(\pi \cos 2x) = -I_{3}$$

54. 
$$f_n = f_{n-1} + f_{n-2}, f_1 = 2, f_2 = 3$$