SAKSHAM
JEE | NEET | GUJCET | KVPY | FOUNDATION
LEARN TOGETHER, GROW FOREVER

Date: 24.01.2023

Time: 3 hrs | CLASS - 12 (MATHEMATICS) - GSEB | Total Marks: 100

Section A

- **Choose correct answer from the given options. [Each carries 1 Mark]** **[24]**

1. $\int_0^{100\pi} \sqrt{1 - \cos 2x}\, dx$ = ...

(A) $100\sqrt{2}$ (B) $200\sqrt{2}$ (C) $50\sqrt{2}$ (D) None of these

2. $\int \sqrt{\frac{x}{a^3 - x^3}}\, dx$ = ... + c.

(A) $\sin^{-1}\left(\frac{x}{a}\right)^{\frac{3}{2}}$ (B) $\frac{3}{2}\sin^{-1}\left(\frac{x}{a}\right)^{\frac{3}{2}}$ (C) $\frac{2}{3}\sin^{-1}\left(\frac{x}{a}\right)^{\frac{3}{2}}$ (D) $\frac{2}{3}(a^3 - x^3)^{\frac{3}{2}}$

3. $I = \int_0^1 \frac{\sin x}{\sqrt{x}}\, dx$ and $J = \int_0^1 \frac{\cos x}{\sqrt{x}}\, dx$ then which of the following statement is true ?

(A) $I > \frac{2}{3}$ and $J > 2$ (B) $I < \frac{2}{3}$ and $J < 2$ (C) $I < \frac{2}{3}$ and $J > 2$ (D) $I > \frac{2}{3}$ and $J < 2$

4. $\int \frac{x^3}{x+1}\, dx$ = ... + C

(A) $x + \frac{x^2}{2} + \frac{x^3}{3} - \log|1 - x|$ (B) $x + \frac{x^2}{2} - \frac{x^3}{3} - \log|1 - x|$

(C) $x - \frac{x^2}{2} - \frac{x^3}{3} - \log|1 + x|$ (D) $x - \frac{x^2}{2} + \frac{x^3}{3} - \log|1 + x|$

5. $\int \frac{a\,dx}{b + ce^x}$ = ... + c

(A) $\frac{a}{b}\log\left(\frac{e^x}{b + ce^x}\right)$ (B) $\frac{a}{b}\log\left(\frac{b + ce^x}{e^x}\right)$ (C) $\frac{b}{a}\log\left(\frac{e^x}{b + ce^x}\right)$ (D) $\frac{b}{a}\log\left(\frac{b + ce^x}{e^x}\right)$

6. Statement - 1 : If $x > 0$, $x \neq 1$ then $\int \left[\log_x e - (\log_x e)^2\right] dx = x \log_x e + c$

Statement - 2 : $\int e^x [f(x) + f'(x)]\, dx = e^x f(x) + c$ and $e^t = x \Leftrightarrow t = \log x$

(A) Statement 1 and 2 are true. Statement 2 is explanation of statement 1.
(B) Statement 1 and 2 are true. Statement 2 is not explanation of statement 1.
(C) Statement 1 is true.
(D) Statement 2 is true. But statement 1 is not true.

7. If $\int f(x)dx = F(x)$ then $\int x^3 f(x^2)dx$ = ...

(A) $\frac{1}{2}\left[x^2 \{F(x)\}^2 - \int \{F(x)\}^2\, dx\right]$ (B) $\frac{1}{2}\left[x^2\, F(x)^2 - \int F(x^2)\, d(x^2)\right]$

(C) $\frac{1}{2}\left[x^2\, F(x) - \frac{1}{2}\int \{F(x)\}^2\, dx\right]$ (D) None of these

8. Area of the curve bounded by the region $(y - x)^2 = x^3$ and the lines $x = 0$ and $x = 1$ is .... sq. units.

(A) $\frac{3}{5}$ (B) $\frac{4}{5}$ (C) $\frac{2}{5}$ (D) $\frac{1}{5}$

9. The area of the region bounded by the parabola $y^2 = 4ax$ and its latus return is 24 sq. units. Then a = ...

(A) $\pm\frac{3}{2}$ (B) ± 3 (C) ± 6 (D) 9

10. The area of the region bounded by the curves $y = 2^x$ and $y = 2x - x^2$ and $x = 0$, $x = 2$ is ...... Sq. unit.

(A) $\frac{4}{3} - \frac{1}{\log 2}$ (B) $\frac{3}{\log 2} + \frac{4}{3}$ (C) $\frac{4}{\log 2} - 1$ (D) $\frac{3}{\log 2} - \frac{4}{3}$

11. The solution of the equation $(2y - 1)dx - (2x + 3)dx = 0$ is ....

(A) $\frac{2x-1}{2y+3} = k$ (B) $\frac{2y+1}{2x-3} = k$ (C) $\frac{2x+3}{2y-1} = k$ (D) $\frac{2x-1}{2y-1} = k$

12. The solution of differential equation $xdy - ydx = 0$ represents ....

(A) rectangular hyperbola (B) parabola whose vertex is at origin

(C) straight line passing through origin (D) a circle whose centre is at origin

13. The order and degree of the family of curves $y^2 = 2c(x + \sqrt{c})$ are .... respectively.

(A) 1, 2 (B) 1, 1 (C) 1, 3 (D) 2, 2

14. A(1, –2, 4), B(5, –1, 7), C(3, 6, –2) and D(4, 5, –1) are given vectors. then the projection of $\overrightarrow{AB}$ on $\overrightarrow{CD}$ is ....

(A) (1, –1, 1) (B) $\frac{3}{13}(4, 1, 3)$ (C) $(2\sqrt{3}, -2\sqrt{3}, 2\sqrt{3})$ (D) (2, –2, 2)

15. $\bar{a}, \bar{b}$ and $\bar{c}$ are unit vectors. The value of $|\bar{a} - \bar{b}|^2 + |\bar{b} - \bar{c}|^2 + |\bar{c} - \bar{a}|^2$ is not expected ....

(A) 4 (B) 9 (C) 8 (D) 6

16. $\bar{a} = \hat{i} + \hat{j} + \hat{k}$, $\bar{b} = \hat{i} + 3\hat{j} + 5\hat{k}$ and $\bar{c} = 7\hat{i} + 9\hat{j} + 11\hat{k}$ are vectors. The area of the parallelogram whose diagonals are $\bar{a} + \bar{b}$ and $\bar{b} + \bar{c}$ is ....

(A) $4\sqrt{6}$ (B) $\frac{1}{2}\sqrt{21}$ (C) $\frac{\sqrt{6}}{2}$ (D) $\sqrt{6}$

17. Vector $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = \hat{i} + \hat{j} + \hat{k}$. The vector $\vec{c}$ is such that $\vec{a} \times \vec{c} + \vec{b} = 0$ and $\vec{a} \cdot \vec{c} = 4$ then $|\vec{c}|^2$ = ...

(A) 8 (B) $\frac{19}{2}$ (C) 9 (D) $\frac{17}{2}$

18. $\vec{a} = \hat{j} - \hat{k}$ and $\hat{c} = \hat{i} - \hat{j} - \hat{k}$. The vector $\vec{b}$ is such that $\vec{a} \times \vec{b} + \vec{c} = 0$ and $\vec{a} \cdot \vec{b} = 3$ then $\vec{b}$ = ...

(A) $-\hat{i} + \hat{j} - 2\hat{k}$ (B) $2\hat{i} - \hat{j} + 2\hat{k}$ (C) $\hat{i} - \hat{j} - 2\hat{k}$ (D) $\hat{i} + \hat{j} - 2\hat{k}$

19. Lines $L_1 : \frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2}$ $L_2 : \frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}$ then unit vector perpendicular L_1 and L_2 is ....

(A) $\frac{-\hat{i} + 7\hat{j} + 7\hat{k}}{\sqrt{99}}$ (B) $\frac{-\hat{i} - 7\hat{j} + 5\hat{k}}{5\sqrt{3}}$ (C) $\frac{-\hat{i} + 7\hat{j} + 5\hat{k}}{5\sqrt{3}}$ (D) $\frac{7\hat{i} - 7\hat{j} - \hat{k}}{\sqrt{99}}$

20. The perpendicular distance of the point (3, –4, –5) from the line $\frac{x-2}{4} = \frac{y+6}{5} = \frac{z-5}{-3}$ is ....

(A) $\frac{1}{5}\sqrt{1657}$ (B) $\frac{1}{\sqrt{5}}\sqrt{1675}$ (C) $\frac{1}{5}\sqrt{1757}$ (D) $\frac{1}{\sqrt{5}}\sqrt{1667}$

21. If two events A and B are such that P(A') = 0.3, P(B) = 0.5 and P(A ∩ B) = 0.3, then P(B|A ∪ B') is ......

(A) 0.375 (B) 0.32 (C) 0.31 (D) 0.28

22. The maximum value of Z = $x + 3y$ subject to the constraints $2x + y \le 20$, $x + 2y \le 20$, $x \ge 0$, $y \ge 0$ is ....

(A) 10 (B) 60 (C) 40 (D) 30

23. The probability that a student is not a swimmer is $\frac{4}{5}$. The probability that out of 5 students exactly 4 are swimmers is ......

(A) $\left(\frac{1}{5}\right)^3$ (B) $4\left(\frac{1}{5}\right)^4$ (C) ${}_5C_4\left(\frac{4}{5}\right)^4$ (D) $\left(\frac{4}{5}\right)^4$

24. The mean and standard deviation of a random variable X are given by E(X) = 5 and $\sigma_x = 3$ respectively, then

i) $E(X^2)$ = ...

ii) $E[(3X - 2)^2]$ = ...

iii) V(3 − 2X) = ...

(A) (i) 34, (ii) 250, (iii) 36 (B) (i) 34, (ii) 370, (iii) 81

(C) (i) 34, (ii) 370, (iii) 36 (D) (i) 34, (ii) 250, (iii) 81

Section B

- **Write the answer of the following questions. [Each carries 2 Marks]** **[16]**

1. The two adjacent sides of a parallelogram are $2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\hat{i} - 2\hat{j} - 3\hat{k}$. Find the unit vector parallel to its diagonal. Also, find its area.

2. Find the area enclosed by the curve $y = -x^2$ and the straight line $x + y + 2 = 0$.

3. Two dice are tossed once. If number 4 comes on first dice then find probability of an event that sum of numbers obtain on two dice is 8 or more.

4. If the line drawn from the point (–2, –1, –3) meets a plane at right angle at the point (1, –3, 3), then find the equation of the plane.

5. Find $\int_0^{\pi} x \sin x \cos^2 x \, dx$

6. Find a vector of magnitude 5 units, and parallel to the resultant of the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$.

7. Find $\int \frac{\cos(5x) + \cos(4x)}{1 - 2\cos(3x)} dx$

8. In answering a question in the multiple choice test, a student either knows the answer or he guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses the answer will be correct with probability $\frac{1}{4}$. What is the probability that the student knows the answer given that he answered it correctly ?

Section C

- **Write the answer of the following questions. [Each carries 3 Marks]** **[36]**

9. Find the shortest distance between the lines whose vector equations are
$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda\,(\hat{i} - 3\hat{j} + 2\hat{k})$ and $\vec{r} = 4\hat{i} + 5\hat{j} + 6\hat{k} + \mu\,(2\hat{i} + 3\hat{j} + \hat{k})$.

10. If the coordinates of the points A, B, C, D be (1, 2, 3), (4, 5, 7), (–4, 3, –6) and (2, 9, 2) respectively, then find the angle between the lines AB and CD.

11. One kind of cake requires 200g of flour and 25g of fat, and another kind of cake requires 100g of flour and 50g of fat. Find the maximum number of cakes which can be made from 5kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes.

12. Find the foot of perpendicular from the point (0, 2, 3) on the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$. Also find the length of perpendicular.

13. If $y = e^{a\cos^{-1}x}$ show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$. Where $-1 \le x \le 1$.

14. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$ for, $-1 < x < 1$ then prove that, $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$

15. Find the vector equation of the plane passing through the intersection of the planes
$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 6$ and $\vec{r} \cdot (2\hat{i} + 3\hat{j} + \hat{k}) = -5$ and the point (1, 1, 1).

16. If a fair coin is tossed 10 times, find the probability of
(a) exactly six heads
(b) atleast six heads
(c) atmost six heads

17. Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$ which is parallel to the line $2x - y + 9 = 0$.

18. Find the shortest distance between the lines $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$.

19. Solve the following linear programming problem graphically :
Minimise Z = $200x + 500y$
Subject to the constraints
$x + 2y \ge 10$
$3x + 4y \le 24$
$x \ge 0,\ y \ge 0$

20. Solve the following linear programming problem graphically.
Maximize Z = $5x + 3y$ Subject to $3x + 5y \le 15$, $5x + 2y \le 10$, $x \ge 0$, $y \ge 0$.

Section D

- **Write the answer of the following questions. [Each carries 4 Marks]** **[24]**

21. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

22. Find $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\, dx$.

23. Solve the differential equation :

$$(x\,dy - y\,dx)\,y\sin\left(\frac{y}{x}\right) = (y\,dx + x\,dy)\,x\cos\left(\frac{y}{x}\right).$$

24. The volume of spherical balloon being inflated changes at a constant rate. If initially its radius is 3 units and after 3 seconds it is 6 units. Find the radius of balloon after t seconds.

25. Find $\int \frac{5x}{(x+1)(x^2+9)}\,dx$

26. Find the particular solution of the differential equation : $\frac{dy}{dx} - \frac{y}{x} + \operatorname{cosec}\left(\frac{y}{x}\right) = 0$, $y = 0$ when $x = 1$