CCT - 50 (JEE ADVANCED)

(PAPER - 2)

DATE: 25.05.2023

TIME: 3HRS

IMPORTANT INSTRUCTIONS

Max Marks: 180

PHYSICS

Section	Question Type	+Ve Marks	- Ve Marks	No.of Qs	Total marks
Sec – I (Q.N : 1 – 6)	Questions with Multiple Correct Choice (partial marking scheme) (+1,0)	+4	-2	6	24
Sec – II (Q.N : 7 – 12)	Questions with Comprehension Type With Numerical value type (3 Comprehensions – 2 + 2 + 2 = 6Q)	+2	0	6	12
Sec – III (Q.N : 13 – 16)	Questions with Comprehension Type (2 Comprehensions – 2 + 2 = 4Q)	+3	−1	4	12
Sec – IV (Q.N : 17 – 19)	Questions with Non-Negative Integer type	+4	0	3	12
Total				**19**	**60**

CHEMISTRY

Section	Question Type	+Ve Marks	- Ve Marks	No.of Qs	Total marks
Sec – I (Q.N : 20 – 25)	Questions with Multiple Correct Choice (partial marking scheme) (+1,0)	+4	-2	6	24
Sec – II (Q.N : 26 – 31)	Questions with Comprehension Type With Numerical value type (3 Comprehensions – 2 + 2 + 2 = 6Q)	+2	0	6	12
Sec – III (Q.N : 32 – 35)	Questions with Comprehension Type (2 Comprehensions – 2 + 2 = 4Q)	+3	−1	4	12
Sec – IV (Q.N : 36 – 38)	Questions with Non-Negative Integer type	+4	0	3	12
Total				**19**	**60**

MATHS

Section	Question Type	+Ve Marks	- Ve Marks	No.of Qs	Total marks
Sec – I (Q.N : 39 – 44)	Questions with Multiple Correct Choice (partial marking scheme) (+1,0)	+4	-2	6	24
Sec – II (Q.N : 45 – 50)	Questions with Comprehension Type With Numerical value type (3 Comprehensions – 2 + 2 + 2 = 6Q)	+2	0	6	12
Sec – III (Q.N : 51 – 54)	Questions with Comprehension Type (2 Comprehensions – 2 + 2 = 4Q)	+3	−1	4	12
Sec – IV (Q.N : 55 – 57)	Questions with Non-Negative Integer type	+4	0	3	12
Total				**19**	**60**

PHYSICS

MAX.MARKS: 60

SECTION - 1 (Maximum Marks : 24)

This section contains **SIX (06)** questions.

Each question has FOUR options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).

For each question, choose the correct option(s) to answer the question.

Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If only (all) the correct option(s) is (are) chosen.

**Partial Marks:** +3 If all the four options are correct but ONLY three options are chosen.

Partial Marks: +2 If three or more options are correct but ONLY two options are chosen, both of which are correct options.

Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.

**Zero Marks :** 0 If none of the options is chosen (i.e. the question is unanswered).

Negative Marks: -2 In all other cases.

1. A thin rod of length 'b' and mass m is suspended horizontally using ideal strings tied to both ends of the rod. The length of the strings is 'a'. The rod is given an initial angular speed ω about its central axis. Let δy be the upwards displacement of rod's center in a small time interval δt and δF be the total increment in the tension forces just after the rod was given the angular speed
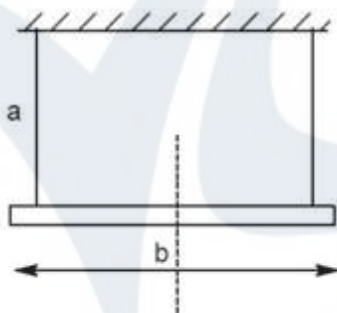

A) $\delta y = \dfrac{b^2}{8a}(\omega\delta t)^2$ B) $\delta y = \dfrac{b^2}{2a}(\omega\delta t)^2$ C) $\delta F = \dfrac{mb^2}{4a}\omega^2$ D) $\delta F = \dfrac{mb^2}{8a}\omega^2$

2. A string of length '2a' has been tied at A and B such that AB = 'a'. The string is passing through a bead C and initially the bead is very close to A. Now the bead is allowed to fall. Let h be the height by which the bead falls when the string becomes taut and v be the speed of the bead just after the string becomes taut. Pick correct option(s):—
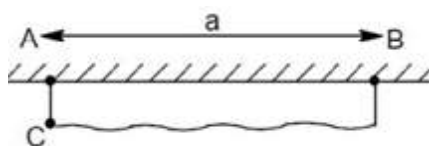

A) v= $\sqrt{3ag}\,Sin(26.5)°$ B) v = $\sqrt{1.5ag}\,Sin(26.5)°$ C) h = $\dfrac{3a}{5}$ D)h = $\dfrac{3a}{4}$

3. Figure shows the initial position of a system of two particles. Given that centre of mass of the system remains at rest and particle A moves in a trajectory given by $\frac{x^2}{a^2}+\frac{y^2}{a^2}=1$. Pick the correct option(s) for the nature of B's trajectory and the coordinates of B at the instant when A was at the position (0, b).
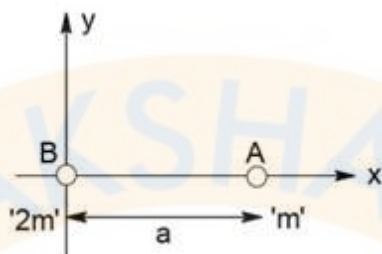

A) Trajectory of B is elliptical

B) Trajectory of B is circular

C) (0, -b/2)

D) (a/2, -b/2)

4. In the figure shown an observer O_1 floats (static) on water surface with ears in air while another observer O_2 is moving upwards with constant velocity $V_1 = V/5$ in water. The source moves down with constant velocity $V_S = V/5$ and emits sound of frequency 'f'. The velocity of sound in air is V and that in water is 4V. For the situation shown in figure.
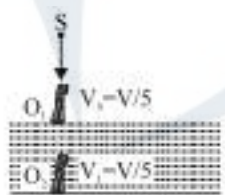

A) The wavelength of the sound received by O_1 is 4V/5f

B) The wavelength of the sound received by O_1 is V/f

C) The frequency of the sound received by O_2 is 21f/16

D) The wavelength of the sound received by O_2 is 16V/5f

5. In the arrangement shown in the figure, initially the block of mass m_1 is in limiting equilibrium. The coefficient of friction between the block of mass m_1 and floor is μ. Now the particle of mass m_2 is given a horizontal velocity v_0. The initial distance of the block of mass m_2 from the pulley is l. The masses of pulley and threads are negligible. Choose the correct option(s):
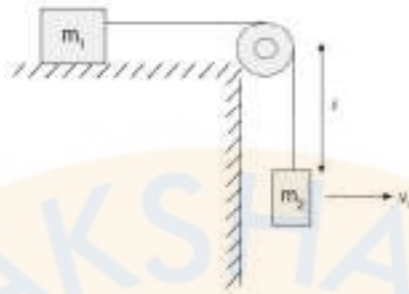
A) Acceleration of m_1 just after the velocity given is $\left(\dfrac{\mu}{\mu+1}\right)\dfrac{v_0^2}{l}$

B) Acceleration of m_1 just after the velocity given is $\left(\dfrac{\mu}{\mu+1}\right)\dfrac{v_0^2}{2l}$

C) Radius of curvature of path of m_2 jsut after the velocity given is $l(1+\mu)$

D) Radius of curvature of path of m_2 just after the velocity given is $2l(1+\mu)$

6. In the arrangement shown in figure. Gas is thermally insulated. An ideal gas is filled in the cylinder having pressure P_0 (> atmospheric pressure P_a). Spring of force constant k is initially unstretched. Piston of mass m and area of cross-section S is frictionless. If piston rises up a distance x_0 slowly, then
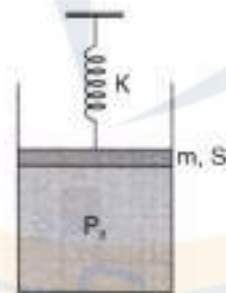
A) final pressure of the gas is $P_a + \dfrac{k x_0}{S} + \dfrac{mg}{S}$

B) work done by the gas is $\dfrac{1}{2}k x_0^2 + mgx_0$

C) decrease in internal energy of the gas is $\dfrac{1}{2}k x_0^2 + mgx_0 + P_a S x_0$

D) decrease in internal energy of the gas is $\dfrac{1}{2}k x_0^2 + mgx_0$

This section contains **TWO (3)** Paragraphs. Based on each paragraph, there are **2** questions.
The answer to each question is a **NUMERICAL VALUE**.
For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter answer. If the numerical value has more than two decimal places **truncate/round- off** the value to **TWO** decimal places.
Answer to each question will be evaluated <u>according to the following marking scheme:</u>
Full Marks: +2 If ONLY the correct numerical value is entered as answer.
Zero Marks: 0 In all other cases.

Question Stem for Question Nos. 7 and 8:

A plane pressure pulse triangular in shape approaches a rigid wall along normal at a speed of 400 m/s. At time t = 0, situation is shown in the figure. The peak pressure is 100 P. By the wall pulse gets reflected and pressure near the wall gets doubled. Height of the wall is 2m and width is also 2 m. A detector on the wall records a minimum excess pressure of 16 pascal.
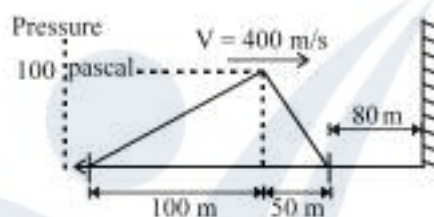
The time for which detector detects the pulse after detecting it for the first time is

$T \times 10^{-2}$ (in sec) and the maximum force (in N) applied by the pulse on the wall is F

7.   Find T

8.   Find F

Question Stem for Question Nos. 9 and 10:

Let us identify the system as the spacecraft and the Earth but not the portion of the fuel in the spacecraft that we use to change the orbit. In a given orbit, the mechanical energy of the spacecraft-Earth system is given by $-\dfrac{GMm}{2r}$. This energy includes the kinetic energy of the spacecraft and the potential energy associated with the gravitational force between the spacecraft and the Earth. If the rocket engines are fired, the thrust force moves the spacecraft through a displacement. As a result, the mechanical energy of the spacecraft-Earth system increases. The spacecraft has a new higher energy but is constrained to be in an orbit that includes the original starting point. It cannot be in a higher energy circular orbit having a larger radius because this orbit would not contain the starting point. The only possibility is that the orbit is elliptical as shown in the figure with $E = -\dfrac{GMm}{2a}$. Above equation gives the energy of the spacecraft-Earth system for an elliptical orbit where 'a' is semi major axis. Thus if we know the new energy of the orbit, we can find the semi-major axis of the elliptical orbit. Conversely, if we know the semi-major axis of an elliptical orbit we would like to achieve, we can calculate how much additional energy is required from the rocket engines.
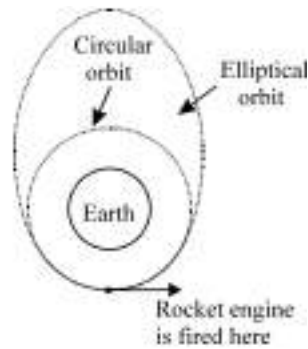
A spacecraft is moving in a circular orbit around the Earth (Radius 6400 Km), at a height of 300 km from the surface. To place the spacecraft in an elliptical orbit, the magnitude of the mechanical energy of the spacecraft Earth system is decreased by 10.0%.

If the spacecraft-earth system had initial energy ($-E_0$) then the total mechanical energy of the system after firing the rocket will be -x E_0. Semi major axis of the new elliptical orbit is 10a (in Km) and the maximum height of the spacecraft above the surface of Earth is 10b (in Km).

9. Find x

10. Find a+b

Question Stem for Question Nos. 11 and 12:

A bicycle has pedal rods of length 16 cm connected to a sprocketed disc of radius 10 cm. The bicycle wheels are 70 cm in diameter and the chain runs over a gear of radius 4 cm. The speed of the cycle is constant and the cyclist applies 100 N force that is always perpendicular to the pedal rod, as shown. Assume tension in the lower part of chain negligible. The cyclist is peddling at a constant rate of two revolutions per second. Assume that the force applied by other foot is zero when one foot is exerting 100 N force. Neglect friction within cycle parts and the rolling friction.

The tension in the upper portion of the chain (in N) is equal to P and the net force of the friction (in N) on the rear wheel due to the road is Q (Round off to two decimal places). The power delivered by the cyclist(in Watt) is equal $W \times \pi$:
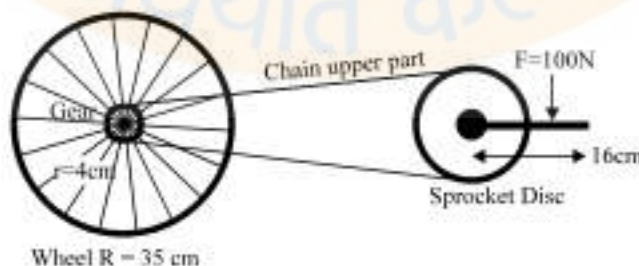
11. Find (P+Q)

12. Find W

SECTION - 3 (Maximum Marks : 12)

This section contains **TWO (2)** Paragraphs. Based on each paragraph, there are **2** questions.

Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer. For each question, choose the option corresponding to the correct answer.

Answer to each question will be evaluated <u>according to the following marking scheme:</u>

**Full Marks:** +3 If ONLY the correct numerical value is entered as answer.
Zero Marks: 0 In all other cases.
Negative Marks: −1 In all other cases.

Question Stem for Question Nos. 13 and 14:

The capacitor C_1 in the figure shown carries a charge q_0 initially. When the switches S_1 and S_2 are closed, capacitor C_1 is connected in series to a resistor R and a second capacitor C_2, which is initially uncharged. Given $C = \dfrac{C_1 C_2}{C_1 + C_2}$.
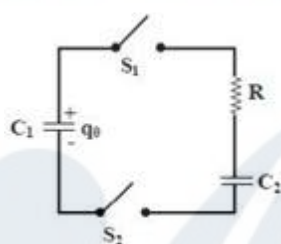
13. The charge flown through wire as a function of time t is

A) $q_0 e^{-\frac{t}{RC}} + \dfrac{C}{C_2} q_0$

B) $q_0 \dfrac{C}{C_1}(1 - e^{-\frac{t}{RC}})$

C) $q_0 \dfrac{C}{C_1} e^{-\frac{t}{RC}}$

D) $q_0 e^{-\frac{t}{RC}}$

14. The total heat dissipated in the circuit during the discharging process of C_1 is

A) $\dfrac{q_0^2}{2C_1^2} \times C$

B) $\dfrac{q_0^2}{2C}$

C) $\dfrac{q_0^2 C_2}{2C_1^2}$

D) $\dfrac{q_0^2}{2C_1 C_2}$

Question Stem for Question Nos. 15 and 16:

The fact that a changing magnetic flux produces an electric field is basic to the operation of many high energy particle accelerators. Since the principle was first successfully applied to the acceleration of electrons (or β particles) in a device called the betatron, this method of acceleration is often given that name. The general idea involved is shown in figure.
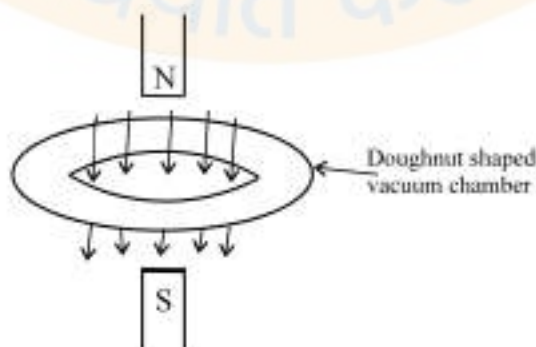
An electromagnet is used to produce a changing flux through a circular loop defined by the doughnut shaped vacuum chamber. We see that there will be an electric field E along the circular length of the doughnut, i.e. circling the magnet poles, given by , $2\pi aE = \dfrac{d\varphi}{dt}$, where 'a' is the radius of the doughnut. Any charged particle inside the vacuum chamber will experience a force qE and will accelerate. Ordinarily, the charged particle would shoot out of the vacuum chamber and becomes lost. However, if the magnetic field at the position of the doughnut is just proper to satisfy the relation, Centripetal force = magnetic force or $\dfrac{mv^2}{a} = q\,vB$, then the charge will travel in a circle within the doughnut. By proper shaping of the magnet pole pieces, this relation can be satisfied. As a result, the charge will move at high speed along the loop within the doughnut. Each time it goes around the loop, it has, in effect, fallen through a potential difference equal to the induced emf, namely $\dfrac{d\varphi}{dt}$. Its energy after 'n' trips around the loop will be q (nε).

15. Working of betatron is not based upon which of the following theories.

 A) changing magnetic flux induces electric field

B) charged particles at rest can be accelerated only by electric fields

C) magnetic fields can apply a force on moving charges which is perpendicular to both magnetic field and motion of the particle

D) β particles are emitted in radioactive decay process.

16. Magnetic field which keeps the particles in circular path must:

A) remain a constant everywhere.

B) increase gradually which is proportional to kinetic energy of the particle

C) increase gradually which is proportional to speed of the particle

D) none of these

SECTION - 4 (Maximum Marks : 12)

This section contains **THREE (03)** questions. The answer to each question is a **NON-NEGATIVE INTEGER**
For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter answer.
Answer to each question will be evaluated <u>according to the following marking scheme:</u>
Full Marks: +4 If ONLY the correct numerical value is entered as answer.
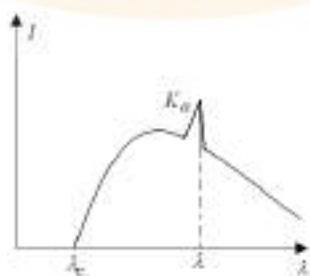**Zero Marks:**   0 In all other cases.

17. A converging lens (focal length f) is broken in two equal pieces and placed at 60 cm as shown along with the object. It is found that real images are formed at the same place and ratio of image heights is 9:1, if the value of f is $\left(22 + \dfrac{1}{x}\right)$ cm then value of x is

    (Consider the formation of image due to single refraction from lens)
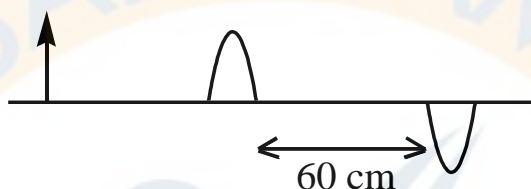
18. A tube with thin but uniform cross section has two arms, one straight, other shaped as a semicircle of radius r. Initially both arms carry an ideal fluid upto a height R. Now the equilibrium is disturbed by pushing the fluid in the left arm by a small amount. Fluid is then released and allowed to oscillate. Neglect any friction or viscous forces. If the time period of oscillations is found to be $T = \pi \sqrt{\dfrac{nR}{3g}(\pi + n)}$; find the integer value n

19. Given X- ray spectrum is for a Coolidge tube having accelerating potential V. If accelerating potential is decreased to $\dfrac{V}{4}$ and atomic number of target metal changed from Z to $\dfrac{Z}{k}$, then $\Delta\lambda = (\lambda - \lambda_c)$ becomes four times. Find k. (neglect screening effect)

CHEMISTRY

MAX.MARKS: 60

SECTION - 1 (Maximum Marks : 24)

This section contains **SIX (06)** questions.

Each question has FOUR options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).

For each question, choose the correct option(s) to answer the question.

Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If only (all) the correct option(s) is (are) chosen.

Partial Marks: +3 If all the four options are correct but ONLY three options are chosen.

Partial Marks: +2 If three or more options are correct but ONLY two options are chosen, both of which are correct options.

Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).

Negative Marks: -2 In all other cases.

20. Extraction of copper from copper pyrite involves:

    A) crushing followed by concentration of the ore by froth-flotation

    B) removal of iron as slag

    C) self-reduction step to produce 'blister copper' following evolution of SO_2

    D) refining of 'blister copper' by carbon reduction

21. Identify the correct statement(s) of the following.

    A) The mineral kinoite ($Ca_2Cu_2Si_3O_{10}.2H_2O$) is a chain silicate.

    B) HF is used as etching agent in glass industry.

    C) In the titration of Fe^{3+} with Sn^{2+}, KCNS can be used as a suitable indicator.

    D) On reaction of red lead with conc. HCl, a golden yellow gas is evolved.

22. Select the correct statements among the following:

    A) Arrhenius theory is NOT applicable to zero order reactions.

    B) If $E_a > 0$, K < A; effective collisions are LESS than available collisions.

    C) If $E_a = 0$, Boltzmann factor =1; effective collisions are LESS than available collisions.

   D) Reactions with higher value of E_a are more sensitive towards rise in temperature.

23. The radial distribution function P(r) is used to determine the most probable radius, which is used to find the electron in a given orbital. For 1s electron of hydrogen like atom with atomic number Z in a given as

$$\frac{dP}{dr} = \frac{4Z^3}{a_0^3}\left(2r - \frac{2Zr^2}{a_0}\right)e^{-\frac{2Zr}{a_0}}$$

Which of the following statement(s) is/are correct ?

A) At the point of maximum value of radial distribution function $\frac{dP}{dr} = 0$; one antinode is present.

B) Most probable radius of Li^{2+} is $\frac{a_0}{3}$

C) Most probable radius of He^+ is $\frac{a_0}{2}$

D) Most probable radius of H is a_o

24. The correct statement(s) for the following reactions is (are)

Reaction I

H$_3$C———CH$_3$ →(Na, liq. NH$_3$) **[Q]**

[P]

Reaction II

[Q] →(Br$_2$, CHCl$_3$) **[R]**
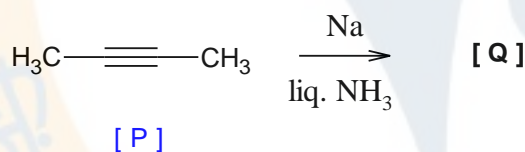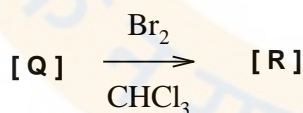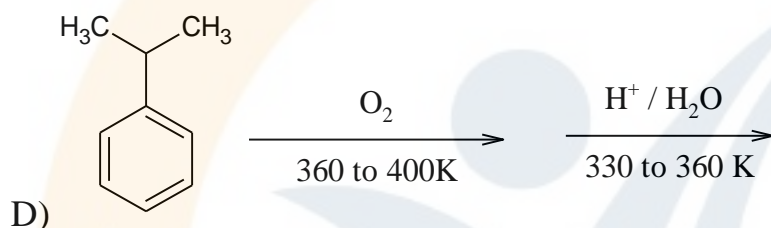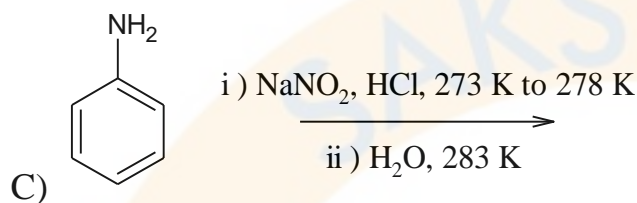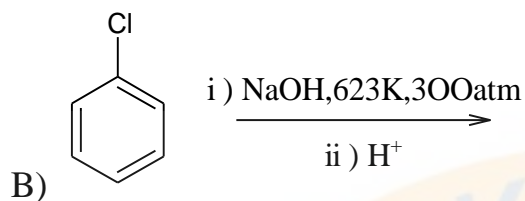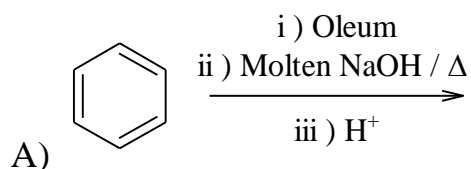
 A) **Reaction I** proceed through free radical (one of the reaction intermediate)

B) **Reaction II** proceed through trans addition

C) Both **Reaction I** and **Reaction II** proceed through cyclic intermediate

D) The dipole moment for both **[Q]** and **[R]** (Consider stable conformer only, if any) is zero

25. How many of the following can give phenol or phenoxide ion as one of the product?

A)
$$\text{benzene} \xrightarrow[\text{iii) } H^+]{\text{i) Oleum} \quad \text{ii) Molten NaOH / } \Delta}$$

B)
$$\text{chlorobenzene} \xrightarrow[\text{ii) } H^+]{\text{i) NaOH,623K,3OOatm}}$$

C)
$$\text{aniline (NH}_2\text{)} \xrightarrow[\text{ii) } H_2O, 283 K]{\text{i) NaNO}_2, HCl, 273 K \text{ to } 278 K}$$

D)
$$\text{isopropylbenzene } (H_3C-CH-CH_3) \xrightarrow[\text{360 to 400K}]{O_2} \quad \xrightarrow[\text{330 to 360 K}]{H^+ / H_2O}$$

SECTION - 2 (Maximum Marks : 12)

This section contains TWO (3) Paragraphs. Based on each paragraph, there are 2 questions.
The answer to each question is a **NUMERICAL VALUE.**
For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter answer. If the numerical value has more than two decimal places **truncate/round- off** the value to **TWO** decimal places.
Answer to each question will be evaluated <u>according to the following marking scheme:</u>
Full Marks: +2 If ONLY the correct numerical value is entered as answer.
Zero Marks: 0 In all other cases.

Question Stem for Question Nos. 26 and 27:

A magnesium ribbon, when burnt in air, left an ash containing MgO and Mg_3N_2. The ash was found to consume 0.6 mol of HCl according to the reactions :

$$MgO + HCl \longrightarrow MgCl_2 + H_2O.$$
$$Mg_3N_2 + HCl \longrightarrow MgCl_2 + NH_4Cl$$
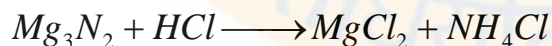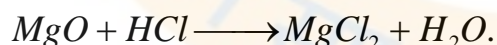
The solution so obtained is treated with excess of NaOH, then 0.1 mol of NH_3 is evolved

$$(NaOH + NH_4Cl \longrightarrow NH_3 + H_2O + NaCl).$$
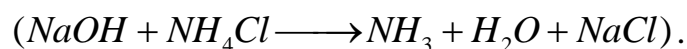
(Atomic mass of Mg=24, N=14, Cl=35.5, H=1, O=16)

26. The mass (in g) of magnesium burnt is.

27. The moles of magnesium nitride reacted is…..

Question Stem for Question Nos. 28 and 29:
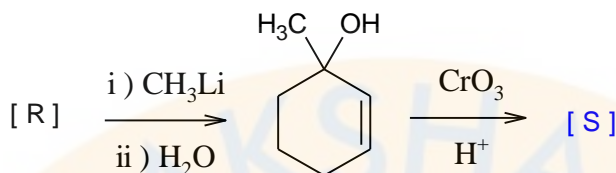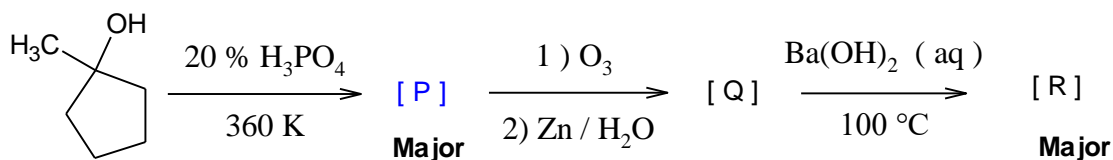
28. The degree of unsaturation (DU) of the **organic product [R]** is ………

29. The number of SP^2 hybridized carbon atoms present in the final major **organic product [S]** is ………

Question Stem for Question Nos. 30 and 31:

A gram atom of carbon reacted with oxygen to from CO and CO_2 at 298K and constant pressure, 53.2 Kcal of heat was released and no carbon remained. Heat of formation of CO and CO_2 are –26 and –94 Kcal/mol respectively.

30. Number of moles of CO formed are

31. Mass of oxygen reacted with carbon in gram is

SECTION - 3 (Maximum Marks : 12)

This section contains **TWO** (**2**) Paragraphs. Based on each paragraph, there are **2** questions.
Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer. For each question, choose the option corresponding to the correct answer.
Answer to each question will be evaluated <u>according to the following marking scheme:</u>
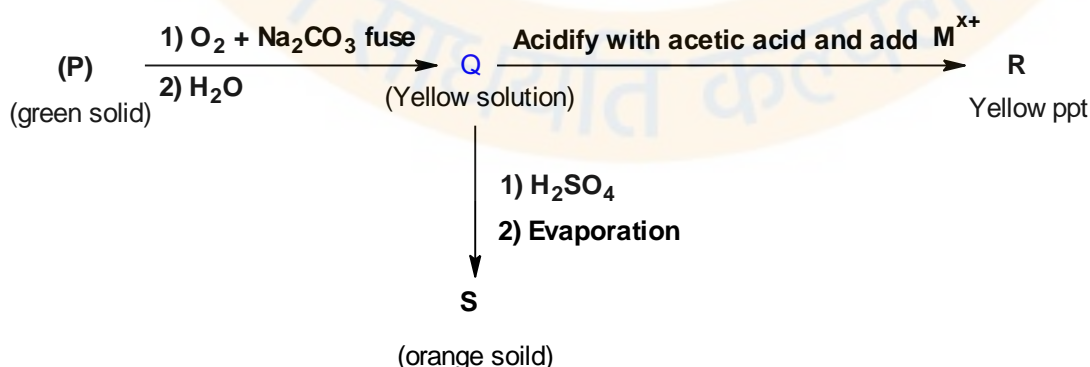**Full Marks:**   +3 If ONLY the correct numerical value is entered as answer.
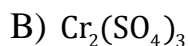**Zero Marks:**   0 In all other cases.
Negative Marks: −1 In all other cases.

Question Stem for Question Nos. 32 and 33:

Answer the following questions on the basis of above reaction sequence.

32. The green solid **P** is:

    A) $FeSO_4.7H_2O$ B) $Cr_2(SO_4)_3$

    C) $NiSO_4.7H_2O$ D) $CuSO_4.5H_2O$

33. The metal ion M^{x+} is:

    A) Ag(I)   B) Hg(I)   C) Pb(II)   D) All

Question Stem for Question Nos. 34 and 35:

Tranquilizers are a class of chemical compounds used for the treatment of stress, and mild or even severe mental diseases. These relieve anxiety, stress, irritability or excitement by inducing a sense of well-being

Noradrenaline is one of the neurotransmitters that plays a role in mood changes. If the level of noradrenaline is low for some reason, then the signal-sending activity becomes low, and the person suffers from depression. In such situations, antidepressant drugs are required.

34. Iproniazide is a _____

    A) Antagonistic drug   B) Agonistic drug

    C) Enzyme inhibitor   D) It relaxes smooth muscles

35. Which tranquilizer is synthesised by the condensation of urea and diethyl – 2,2- diethyl malonate in acidic medium?

    A) Serotonin   B) Equanil

    C) Meprobamate   D) Veronal

SECTION - 4 (Maximum Marks : 12)
This section contains **THREE (03)** questions. The answer to each question is a **NON-NEGATIVE INTEGER**
For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter answer.
Answer to each question will be evaluated <u>according to the following marking scheme:</u>
Full Marks: +4 If ONLY the correct numerical value is entered as answer.
Zero Marks: 0 In all other cases.

36. Aq. Solutions of KI and KIO_3 mixed together in presence of HCl to form I_2 and KC*l*. Released I_2 required 100 mL of decimolar hypo solution for complete reduction. Let the number of milli moles of KI and HCl used in the reaction be " x " and " y " respectively. Find the value of (3x – 2y) ?

37. In how many of the following cases, gas is more compressible than an ideal gas ?

$$\left(R = \frac{1}{12} lit\, atm \deg^{-1} mol^{-1} \right)$$

P: Oxygen gas has density of 20 gL^{-1} at 10 atm and 300 K.

Q: Nitrogen gas at 2 atm and 273 K occupy molar volume of 6 L.

R: Gas A at its critical temperature and pressure less than critical pressure.

S: CH_4 gas at 333 K and in low pressure region.

(Boyle's temperature T_B of CH_4 = 60°C)

38. Zingiberene is a fragrant compound isolated from ginger, has the molecular formula $C_{15}H_{24}$ and is known not to contain any triple bond(s). When zingiberene is subjected to catalytic hydrogenation using an excess of hydrogen gas, one mole of zingiberene absorbs three moles of dihydrogen gas and produce a compound with the molecular formula $C_{15}H_{30}$. By using this data find the number of ring(s) is present in zingiberene.

MATHEMATICS

MAX.MARKS: 60

SECTION - 1 (Maximum Marks : 24)

This section contains **SIX (06)** questions.

Each question has FOUR options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).

For each question, choose the correct option(s) to answer the question.

Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If only (all) the correct option(s) is (are) chosen.

Partial Marks: +3 If all the four options are correct but ONLY three options are chosen.

Partial Marks: +2 If three or more options are correct but ONLY two options are chosen, both of which are correct options.

Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).

Negative Marks: -2 In all other cases.

39. If $\tan\theta_1, \tan\theta_2, \tan\theta_3, \tan\theta_4$ are the roots of the equation

    $x^4 - x^3\sin\beta - x^2\cos\beta + x\sin 2\beta + \tan 2\beta = 0$, then which of the following is/are correct (

    where $\theta_1, \theta_2, \theta_3, \theta_4 \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$)

    A) $\tan\theta_1 + \tan\theta_2 + \tan\theta_3 + \tan\theta_4 = 0$ if $\beta = 10^5\pi$

    B) $\tan\theta_1 + \tan\theta_2 + \tan\theta_3 + \tan\theta_4 = 0$ if $\beta = 11^5\pi$

    C) $\tan\theta_1 \tan\theta_2 \tan\theta_3 \tan\theta_4 = 0$ if $\beta = 2^5\pi$

    D) $\tan\theta_1 \tan\theta_2 \tan\theta_3 \tan\theta_4 = 0$ if $\beta = 5^{11}\pi$

40. Suppose x is a real number such that $\sin\left(1 + \cos^2 x + \sin^4 x\right) = \dfrac{13}{14}$, then the value of

    $\cos\left(1 + \cos^4 x + \sin^2 x\right)$ can be

    A) $\dfrac{3\sqrt{3}}{14}$

    B) $-\dfrac{3\sqrt{3}}{14}$

    C) $-\dfrac{\sqrt{3}}{4}$

    D) $\dfrac{\sqrt{3}}{4}$

41. For every positive real number x, let $g(x) = \lim\limits_{r\to 0}\left((x+1)^{r+1} - x^{r+1}\right)^{\frac{1}{r}}$ then which of the

    following is/are correct?

    A) g(x) is differentiable for all $x > 0$

    B) $\lim\limits_{x\to\infty}\dfrac{g(x)}{x}$ is an integer

    C) $\lim\limits_{x\to\infty}\dfrac{g(x)}{x}$ is a rational number

    D) $\lim\limits_{x\to\infty}\dfrac{g(x)}{x}$ is an irrational number

42. Let S be the set of points (a,b) with $0 \le a,b \le 1$ such that the equation

$x^4 + ax^3 - bx^2 + ax + 1 = 0$ has at least one real root, then which of the following is /are correct?

A) Minimum value of $a^2 + b^2$ is $\dfrac{3}{5}$

B) Minimum value of $a^2 + b^2$ is $\dfrac{4}{5}$

C) Area of the graph of S is $\dfrac{1}{4}$

D) Area of the graph of S is $\dfrac{2}{5}$

43. Let $S_k = \displaystyle\sum_{j=0}^{k} 2^{k-j} \binom{k+j}{j}$; $k \in W$, $\dbinom{n}{r}$ represents ^nC_r and

$\dbinom{n}{r} + \dbinom{n}{r-1} = \dbinom{n+1}{r}$, $S_0 = 1$, $\left(where\ W = \{0,1,2,3,.........\}\right)$

Then which of the following is/ are correct?

A) $S_{100} = 4S_{98}$

B) $S_{100} = 16S_{98}$

C) $\displaystyle\sum_{k=1}^{100} S_k = \dfrac{2}{3}\left(2^{100} - 1\right)$

D) $\displaystyle\sum_{k=1}^{200} S_k = \dfrac{4}{3}\left(2^{400} - 1\right)$

44. Which of the following inequalities is/are true?

A) $\displaystyle\int_0^1 x\tan x\,dx \ge \dfrac{2}{5}$

B) $\displaystyle\int_0^1 x^2\cos x\,dx \le \dfrac{1}{3}$

C) $\displaystyle\int_0^1 x^3 \sin x\,dx \ge \dfrac{3}{20}$

D) $\displaystyle\int_{-1}^0 x\sin x\,dx \ge \dfrac{3}{10}$

This section contains **TWO (3)** Paragraphs. Based on each paragraph, there are **2** questions.
The answer to each question is a **NUMERICAL VALUE**.
For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter answer. If the numerical value has more than two decimal places **truncate/round- off** the value to **TWO** decimal places.
 Answer to each question will be evaluated <u>according to the following marking scheme:</u>
Full Marks: +2 If ONLY the correct numerical value is entered as answer.
Zero Marks: 0 In all other cases.

Question Stem for Question Nos. 45 and 46:

Let A be a matrix of order 3×3 with real entries and satisfying the condition

$$A^{2019} + A = \begin{pmatrix} 2 & 2 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{pmatrix}, \text{ then (Where tr(A) = Trace of A, } |A| = \text{determinant of A)}$$

45. $tr(A^{2019})$ is equal to

46. $|A|$ is equal to

Question Stem for Question Nos. 47 and 48:

Suppose $a_1, a_2, a_3, \ldots, a_{100}$ are positive real numbers such that $a_k = \dfrac{k a_{k-1}}{a_{k-1} - (k-1)}$

for $k = 2, 3, \ldots, 100$. Given that $a_{20} = a_{23}$. Then

47. a_{100} is equal to

48. a_{23} is equal to

Question Stem for Question Nos. 49 and 50:

A certain function f has the properties that $f(3x) = 3f(x)$ for all positive real values of

x, and that $f(x) = 1 - |x-2|$ for $1 \le x \le 3$, N be the smallest positive number that

satisfies $f(N) = f(2001)$, then

49.   Sum of digits of N is equal to_____

50.   Number of positive integral divisors of N is equal to_____

This section contains **TWO (2)** Paragraphs. Based on each paragraph, there are **2** questions.
Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer. For each question, choose the option corresponding to the correct answer.
Answer to each question will be evaluated <u>according to the following marking scheme:</u>
Full Marks: +3 If ONLY the correct numerical value is entered as answer.
Zero Marks: 0 In all other cases.
Negative Marks: −1 In all other cases.

Question Stem for Question Nos. 51 and 52:

If $m > 0, n > 0,$ the definite integral $I = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$ depends upon the values of m and n is denoted by $\beta(m,n)$, called the beta function.

e.g. $\int_0^1 x^4(1-x)^5\,dx = \int_0^1 x^{5-1}(1-x)^{6-1}\,dx = \beta(5,6)$ and

$\int_0^1 x^{\frac{5}{2}}(1-x)^{\frac{-1}{2}}\,dx = \int_0^1 x^{\frac{7}{2}-1}(1-x)^{\frac{1}{2}-1}\,dx = \beta\left(\frac{7}{2}, \frac{1}{2}\right).$ Obviously $\beta(m,n) = \beta(n,m).$

51. The integral $\int_0^{\frac{\pi}{2}} \cos^{2m}\theta \sin^{2n}\theta\,d\theta$ is equal to

A) $\dfrac{1}{2}\beta\left(m+\dfrac{1}{2}, n+\dfrac{1}{2}\right)$

B) $2\beta(2m, 2n)$

C) $\beta(2m+1, 2n+1)$

D) None of these

52. If $\int_0^\infty \dfrac{x^{m-1}}{(1+x)^{m+n}}\,dx = k\int_0^\infty \dfrac{x^{n-1}}{(1+x)^{m+n}}\,dx$, then k is equal to (where $m \neq n$)

A) $\dfrac{m}{n}$

B) 1

C) $\dfrac{n}{m}$

D) mn

Question Stem for Question Nos. 53 and 54:

Let $f : R \to R, y = f(x), f(0) = 0, f'(x) > 0$ and $f''(x) > 0, \forall x \in R$. Three points $A(\alpha, f(\alpha)), B(\beta, f(\beta)), C(\gamma, f(\gamma))$ on $y = f(x)$ such that $0 < \alpha < \beta < \gamma$

53. Which of the following is false?

A) $\alpha f(\beta) > \beta f(\alpha)$

B) $\alpha f(\beta) < \beta f(\alpha)$

C) $\gamma f(\beta) < \beta f(\gamma)$

D) $\gamma f(\alpha) < \alpha f(\gamma)$

54. Which of the following is true?

A) $\dfrac{f(\alpha)+f(\beta)}{2} < f\left(\dfrac{\alpha+\beta}{2}\right)$

B) $\dfrac{f(\alpha)+f(\beta)}{2} > f\left(\dfrac{\alpha+\beta}{2}\right)$

C) $\dfrac{f(\alpha)+f(\beta)}{2} = f\left(\dfrac{\alpha+\beta}{2}\right)$

D) $\dfrac{2f(\alpha)+f(\beta)}{3} < f\left(\dfrac{2\alpha+\beta}{3}\right)$

SECTION - 4 (Maximum Marks : 12)

This section contains **THREE (03)** questions. The answer to each question is a **NON-NEGATIVE INTEGER**
For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter answer.
Answer to each question will be evaluated <u>according to the following marking scheme:</u>
Full Marks: +4 If ONLY the correct numerical value is entered as answer.
Zero Marks: 0 In all other cases.

55. If the number of nonempty subsets $S \subseteq \{-10, -9, -8, \ldots\ldots, 8, 9, 10\}$ that satisfy

$|S| + \min(S).\max(S) = 0$ is equal to k (Where $|S|$ denotes number of elements in S,

$\min(S) =$ minimum element in S, $\max(S) =$ maximum element in S),then

$\left[\dfrac{k}{70}\right] =$ ___ (where [.] represents G.I.F)

56. There are 800 marbles in a bag. Each marble is colored with one of 100 colors, and there are eight marbles of each color. Agastya draws one marble at a time from the bag, without replacement, until he gets eight marbles of the same color, and then he immediately stops. Suppose Agastya has not stopped after drawing 699 marbles. The

probability that he stops immediately after drawing the 700th marble is, $\dfrac{m}{n}$ then $n-m$

is equal to (where m ,n are positive integers which are relatively prime)

57. If $\displaystyle\int_0^1\left(\sum_{n=1}^{\infty}\dfrac{[2^n x]}{3^n}\right)dx = \dfrac{p}{q}$ then $p+q$ is equal to (where p,q are positive integers which are

relatively prime and [.] denotes greatest integer function)