CCT - 50 (JEE ADVANCED)

(PAPER - 1)

DATE: 25.05.2023

TIME: 3HRS

IMPORTANT INSTRUCTIONS

Max Marks: 180

PHYSICS

Section	Question Type	+Ve Marks	- Ve Marks	No.of Qs	Total marks
Sec – I (Q.N : 1 – 4)	Questions with Single Correct Options	+3	-1	4	12
Sec – II (Q.N : 5 – 10)	Questions with Comprehension Type With Numerical value type (3 Comprehensions – 2 + 2 + 2 = 6Q)	+2	0	6	12
Sec – III (Q.N : 11 – 16)	Questions with Multiple Correct Choice (partial marking scheme) (+1,0)	+4	-2	6	24
Sec – IV (Q.N : 17 – 19)	Questions with Non-Negative Integer type	+4	0	3	12
Total				**19**	**60**

CHEMISTRY

Section	Question Type	+Ve Marks	- Ve Marks	No.of Qs	Total marks
Sec – I (Q.N : 20 – 23)	Questions with Single Correct Options	+3	-1	4	12
Sec – II (Q.N : 24 – 29)	Questions with Comprehension Type With Numerical value type (3 Comprehensions – 2 + 2 + 2 = 6Q)	+2	0	6	12
Sec – III (Q.N : 30 – 35)	Questions with Multiple Correct Choice (partial marking scheme) (+1,0)	+4	-2	6	24
Sec – IV (Q.N : 36 – 38)	Questions with Non-Negative Integer type	+4	0	3	12
Total				**19**	**60**

MATHS

Section	Question Type	+Ve Marks	- Ve Marks	No.of Qs	Total marks
Sec – I (Q.N : 39 – 42)	Questions with Single Correct Options	+3	-1	4	12
Sec – II (Q.N : 43 – 48)	Questions with Comprehension Type With Numerical value type (3 Comprehensions – 2 + 2 + 2 = 6Q)	+2	0	6	12
Sec – III (Q.N : 49 – 54)	Questions with Multiple Correct Choice (partial marking scheme) (+1,0)	+4	-2	6	24
Sec – IV (Q.N : 55 – 57)	Questions with Non-Negative Integer type	+4	0	3	12
Total				**19**	**60**

PHYSICS

MAX.MARKS: 60

SECTION – 1 (Maximum Marks: 12)

This section contains **SIX (04)** questions.
Each question has FOUR options for correct answer(s). **ONLY ONE** of these four option is the correct answer.
For each question, choose the correct option corresponding to the correct answer.
Answer to each question will be evaluated <u>according to the following marking scheme</u>:
**Full Marks :**   +3 If only the correct option is chosen.
Zero Marks: 0 If none of the option is chosen.(i.e the question is un answered)
Negative Marks: -1 In all other cases.

1. Figure shows a convex lens cut symmetrically into two equal halves and separated laterally by a distance h. A point object O placed symmetrically at a distance 30cm, from the lens halves, within the focus, forms two images separated by a distance d. A plot of d versus h is shown in figure. Focal length of the lens is :-
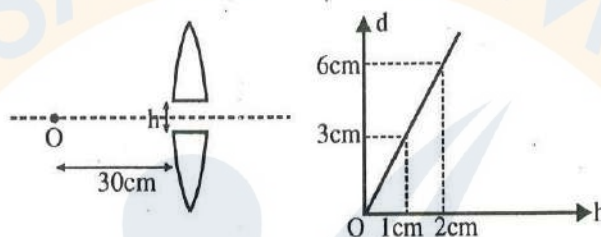
     A) 22.5 cm   B) 40cm   C) 45cm   D) 20cm

2. A point charge q is kept at a point $\left(a, 2a, \dfrac{a}{2}\right)$ in the space. Consider a triangular plane surface whose co-ordinates of vertices are $(a, 2a, 0), \left(\dfrac{3a}{2}, 2a, 0\right)$ and $\left(\dfrac{3a}{2}, \dfrac{5a}{2}, 0\right)$. The electric flux passing through the considered triangular surface is

     A) $\dfrac{q}{12\varepsilon_0}$ B) $\dfrac{q}{24\varepsilon_0}$ C) $\dfrac{q}{48\varepsilon_0}$ D) $\dfrac{q}{96\varepsilon_0}$

3. A uniform circular disc of radius 'R' is rolling without slipping on a rough horizontal surface with a constant acceleration 'a'. Then the radius of curvature of trajectory of point 'A' of the disc relative to the ground at the given instant (when A is at same horizontal level as that of the centre of disc) as shown in the figure is
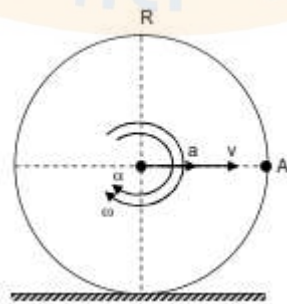
     A) $4R$ B) $2\sqrt{2}R$ C) $2R$ D) $\sqrt{2}R$

4. A nucleus with mass number 224 initially at rest emits an α particle. If the Q value of the reaction is 8 MeV and only one photon of energy 2.4 MeV is subsequently emitted during the emission of the α particle, the kinetic energy (in MeV) of the α particle is

A) 5.0   B) 4.5   C) 5.5   D) 2.0

SECTION - 2 (Maximum Marks : 12)

This section contains **TWO** (**3**) Paragraphs. Based on each paragraph, there are **2** questions.

The answer to each question is a **NUMERICAL VALUE.**

For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter answer. If the numerical value has more than two decimal places **truncate/round- off** the value to **TWO** decimal places.

Answer to each question will be evaluated according to the following marking scheme:

Full Marks: +2 If ONLY the correct numerical value is entered as answer.

Zero Marks: 0 In all other cases.

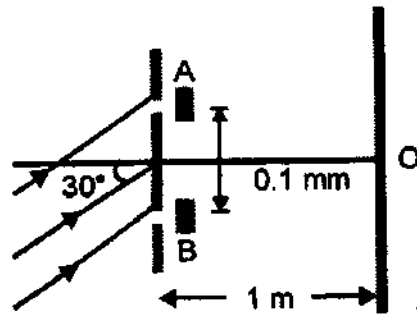**Question Stem for Question Nos. 5 and 6:**

A homogeneous bar of mass M rotates uniformly in alternating directions to and fro around the horizontal axis going through its center, while a ball of mass m bounces between the ends of the bar in vertical plane as shown in the figure. The angular velocity of the bar is $\pm\dfrac{2\pi}{3}$ rad sec^{-1} and the time period of the periodic motion of the ball is 2 second. (The mechanical loss and the duration of the collisions are negligible.)
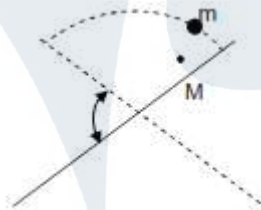
5. The velocity (in m/s) of the ball at the highest point of its orbit is $\alpha\sqrt{3}$. Then α is _____

6. The length of the bar is (in meter) $\beta\sqrt{3}$. Then β is _____

Question Stem for Question Nos. 7 and 8:

In a YDSE a parallel beam of light of wavelength 6000 $\overset{\circ}{A}$ is incident on slits at angle of incidence 30° as shown in fig. A and B are two thin transparent films each of refractive index 1.5. Thickness of A is 20.4 μm. Light coming through A and B have intensities I and 4I respectively on the screen. Intensity at point O which is symmetric relative to the slits is 3I. The central maxima is above O

7.   The maximum thickness of 'B' is (in μm)_____

8.   The ratio of maximum to minimum intensity on the screen is _____

Question Stem for Question Nos. 9 and 10:

A uniform rod AB of mass M is attached to a hinge at one end A, and released from rest from the horizontal position. The rod rotates about A, and when it reaches the vertical position the rod strikes a sphere of mass m and radius r initially at rest on the smooth horizontal surface as shown in the adjacent figure. The impact is along the horizontal direction and perfectly elastic.

At the moment of impact the lowest end of the rod is very close to the smooth horizontal surface. After the impact, the sphere moves along the horizontal and the rod, subsequently rises to a maximum angle of 60° with the vertical. The length of the rod equals $\sqrt{2}r$. (r = $\frac{6\sqrt{2}}{10}$ m)

Taking into account the information given above, find
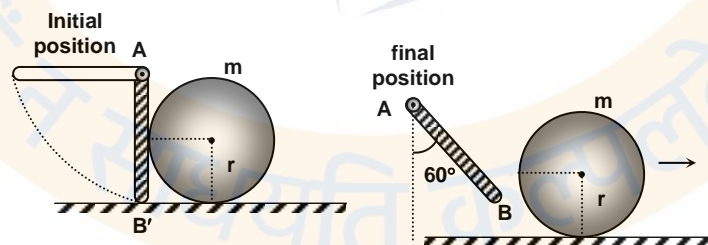
9.   The ratio M/m is _____

10.   The speed of the sphere just after collision is (in m/s)_____

This section contains **SIX (06)** questions.

Each question has FOUR options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).

For each question, choose the correct option(s) to answer the question.

Answer to each question will be evaluated according to the following marking scheme:

Full Marks :   +4 If only (all) the correct option(s) is (are) chosen.

Partial Marks: +3 If all the four options are correct but ONLY three options are chosen.

Partial Marks: +2 If three or more options are correct but ONLY two options are chosen, both of which are correct options.

Partial Marks :   +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.

Zero Marks :   0 If none of the options is chosen (i.e. the question is unanswered).

Negative Marks: -2 In all other cases.

11. A planet is a homogeneous ball of radius 'R' having mass 'M'. It is surrounded by a dense atmosphere having density $\rho = \dfrac{\sigma_0}{r}$. Where σ_0 is constant and r is distance from the centre of planet. Then

A) If $\sigma_0 = \dfrac{M}{2\pi R^2}$ then acceleration due to gravity is constant throughout the atmosphere of planet

B) If $\sigma_0 = \dfrac{M}{\pi R^2}$ then acceleration due to gravity is constant throughout the atmosphere of planet

C) For any value of σ_0, acceleration due to gravity can be constant throughout the atmosphere of planet

D) If $\sigma_0 = \dfrac{M}{2\pi R^2}$ then the weight experienced by a body of mass 'm' at altitude $\dfrac{R}{2}$ from the surface of planet will be $\dfrac{GMm}{R^2}$

12.   Consider a sphere of radius R with a point source of power 'P' inside it. At steady state, temperature of surface of sphere is T_0 . Now, the sphere is surrounded by another spherical shell of radius 2R. (Assume no other radiation from outside and all surfaces are black body). Select correct alternative(s):

A) Before covering power (P) emitted from the surface is $4\pi\sigma T_0^4 R^2$

B) Temperature of inner sphere after covering is $\dfrac{T_0}{\sqrt{2}}$.

C) Temperature of outer sphere in steady state is $\dfrac{T_0}{\sqrt{2}}$.

D) Temperature of inner sphere in steady state after covering is T_0

13. In the circuit shown, initially only switch-1 is closed for a long time. Then switch-2 is also closed. Choose the correct option(s).
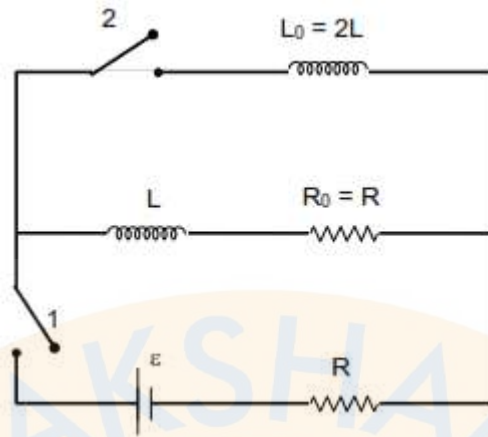

A) Current through the inductor 'L_0' in the steady state after closing the switch-2 will be $\dfrac{\varepsilon}{R}$

B) Current through the inductor 'L_0' in the steady state after closing the switch-2 will be $\dfrac{\varepsilon}{2R}$

C) The total charge $\left(\dfrac{5\varepsilon L}{2R^2}\right)$ will flow through the resistor 'R_0' after closing the switch-2.

D) The total charge $\left(\dfrac{3\varepsilon L}{2R^2}\right)$ will flow through the resistor 'R_0' after closing the switch-2.

14. A bar is placed with slight offset l_0 on another identical bar that rests on a horizontal frictionless floor. Coefficient of friction between the bars is μ. Initially both the bars are moving towards a wall with a velocity u. The wall is parallel to the front face of the bars. The collision of the bars with walls is perfectly elastic. Then choose the correct option(s).
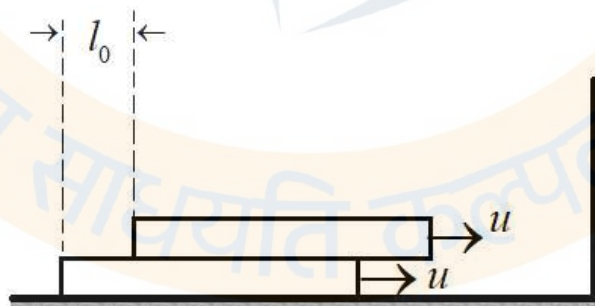

A) The final velocities of both the bars will be same after a long time.

B) The final velocities of both the bars will be zero if $u = \sqrt{\mu g l_0}$.

C) The final velocities of both the bars will be zero if $u = \sqrt{2\mu g l_0}$.

D) The final velocities of both the bars will be $\sqrt{2\mu g l_0}$ if $u = \sqrt{4\mu g l_0}$.

15. For a certain transverse standing wave on a long string, an antinode is formed at x = 0 and next to it, a node is formed at x = 0.10 m. The displacement y(t) of the string particle at x = 0 is shown in figure. Choose the correct option(s)
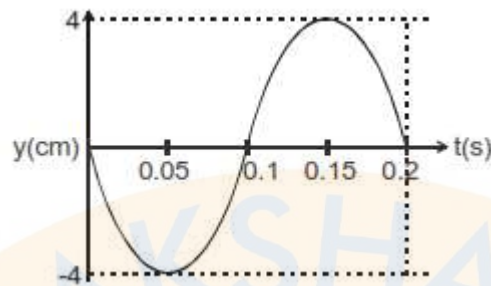
A) Transverse displacement of the particle at x = 0.05m and t = 0.05 s is $-2\sqrt{2}\,cm$

B) Transverse displacement of the particle at x = 0.04 m and t = 0.025 s is $-2\sqrt{2}\,cm$

C) Speed of the travelling waves that interfere to produce this standing wave is 2 m/s.

D) The transverse velocity of the string particle at $x = \dfrac{1}{15}m$ and t = 0.1 s is $20\pi\,cm/s$

16. One mole of a monatomic ideal gas goes through a thermodynamic cycle, as shown in the volume versus temperature (V-T) diagram. The correct statement(s) is/are : (R is the gas constant)
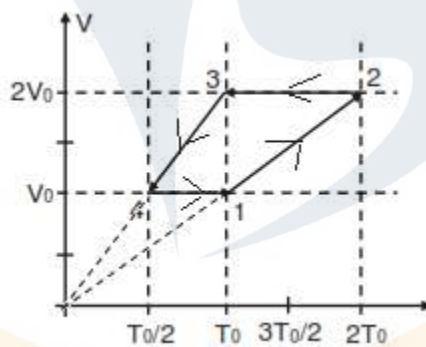
A) Work done in this thermodynamic cycle $(1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 1)$ is $|W| = \dfrac{1}{2}RT_0$

B) The ratio of heat transfer during processes $1 \rightarrow 2$ and $2 \rightarrow 3$ is $\left|\dfrac{Q_{1 \rightarrow 2}}{Q_{2 \rightarrow 3}}\right| = \dfrac{5}{3}$

C) The above thermodynamic cycle exhibits only isochoric and adiabatic processes

D) The ratio of heat transfer during processes $1 \rightarrow 2$ and $3 \rightarrow 4$ is $\left|\dfrac{Q_{1 \rightarrow 2}}{Q_{3 \rightarrow 4}}\right| = \dfrac{1}{2}$

SECTION - 4 (Maximum Marks : 12)
This section contains **THREE (03)** questions. The answer to each question is a **NON-NEGATIVE INTEGER**
For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter answer.
Answer to each question will be evaluated <u>according to the following marking scheme:</u>
Full Marks: +4 If ONLY the correct numerical value is entered as answer.
Zero Marks: 0 In all other cases.

17. Figure shows a system of block, spring and pulley. Spring and pulley are ideal. Surface below the block is smooth. The time period (in second) of small oscillations is $x\pi$. Find x (m = 1kg, k = 5 N/m)
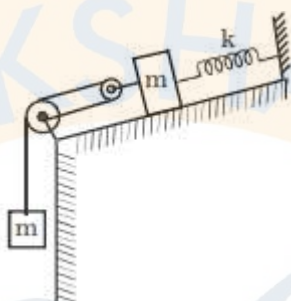
18. Two conducting spherical shells S_1 of radius R and S_2 of radius $2R$ are co-centric. A charge $Q = 20\mu C$ is kept at the centre and another same charge Q is given to S_1 and $-Q$ to S_2. The loss of energy(in Joule) when switch S is closed is 10k. Where k is____(Given $R = 9$ cm)
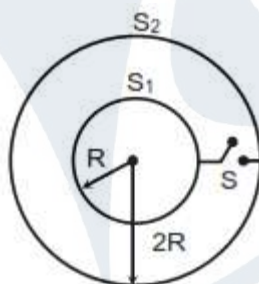
19. Peak emission from a black body at a certain temperature occurs at a wavelength of 4000Å. On increasing its temperature, the total radiation emitted is increased to 256 times. These radiations are allowed to fall on a metal surface. The photoelectrons emitted by peak emission wavelength at higher temperature can be brought to rest by applying a potential which is equivalent to the excitation potential corresponding to the transition from level n = 2 to level n = 4 in the singly ionized Helium atom (obeys Bohr's Model). The work function of the metal (in eV) is x.

Find $\dfrac{x}{2.2}$

[Given E (in eV) $= \dfrac{12400}{\lambda\left(in\,A^0\right)}$]

SECTION – 1 (Maximum Marks: 12)

This section contains **SIX (04)** questions.
Each question has **FOUR** options for correct answer(s). **ONLY ONE** of these four option is the correct answer.
For each question, choose the correct option corresponding to the correct answer.
Answer to each question will be evaluated <u>according to the following marking scheme</u>:
**Full Marks :**   +3 If only the correct option is chosen.
Zero Marks: 0 If none of the option is chosen.(i.e the question is un answered)
Negative Marks: -1 In all other cases.

20. Incorrect statement regarding following reactions is

$$XeF_6 \xrightarrow{\begin{array}{c}+\text{Excess } H_2O\end{array}} 'X' + HF$$
$$XeF_6 \xrightarrow{+2H_2O} 'Y' + HF$$
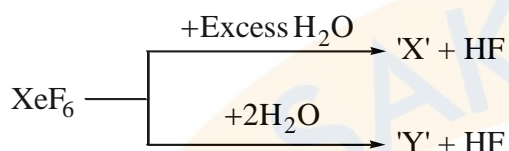
A) 'X' is explosive

B) 'Y' is an oxyacid of xenon

C) Both are example of non-redox reaction

D) XeF_6 can undergo partial hydrolysis

21. K_{sp} of $A(OH)_2 = 4\times10^{-12}$ and that of $B(OH)_3 = 1\times10^{-33}$. Ratio of moles of $A(OH)_2$ and $B(OH)_3$ in 1 litre of their saturated solution will be

A) 8×10^{-17} B) 8×10^{17} C) 4×10^{17} D) 4×10^{-17}

22. In the given compound the order of ease with which hydrogen atom can be abstracted (Homolytic fission) from carbons **I** to **VI** is
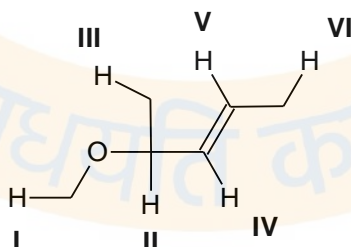
A) $II > VI > IV \square V > I > III$ B) $II > I > VI > III > IV \square V$

C) $II > I > III > VI > IV \square V$ D) $VI > II > I > III > IV \square V$

23. The major products P and Q, formed in the following reactions given below, are

A) and

   P   Q

B) and

   P   Q

C) and

   P   Q

D) and

   P   Q

SECTION - 2 (Maximum Marks : 12)

This section contains **TWO** (**3**) Paragraphs. Based on each paragraph, there are **2** questions.
The answer to each question is a **NUMERICAL VALUE.**

For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter answer. If the numerical value has more than two decimal places **truncate/round- off** the value to **TWO** decimal places.
 Answer to each question will be evaluated <u>according to the following marking scheme:</u>

Full Marks: +2 If ONLY the correct numerical value is entered as answer.

Zero Marks: 0 In all other cases.

Question Stem for Question Nos. 24 and 25:

Sodium azide used in air bags of automobile cars to prevent injuries and death during accidents. Due to sudden shock (collision) sodium azide dissociate into Nitrogen and sodium metal. To prevent the burning of sodium metal formed,the sodium azide is mixed with sodium nitrate (Atomic mass of Na=23, N=14, O=16)

24. How many litres of Nitrogen gas liberated(at NTP) from 130gr of sodium azide ……..

25. Howmuch $NaNO_3$ in grams is required to cosume the sodium metal formed during the dissociation of 130 gr of Sodium azide…..

Question Stem for Question Nos. 26 and 27:

Compound **[P]** is obtained by heating of iodoform (two molecules) with silver powder (excess). Two molecules of **[P]** is passed into a solution of cuprous chloride in ammonium chloride give **[Q]**. Compound **[R]** (Molecular formula C_4H_5Cl) is manufactured by passing **[Q]** into concentrated hydrochloric acid in the presence ofcuprous and ammonium chlorides.

Answer the following two questions by using above data,

26. Number of moles of silver iodide will be produced for the formation of 39 gm of **Compound [P]** is ……

(Given: Atomic mas of H=1, C=12, N=14, Cl=35, Cu=63, Ag=108 and I=127)

27. The correct locant of chlorine (Consider IUPAC name) in the **Compound [R]** is ……

Question Stem for Question Nos. 28 and 29:

A definite amount of a solution " X " undergoing decomposition required 25.6 mL of standard solution of $KMnO_4$ for titration. After 10 and 20 minutes, volume of permanganate solution required were 16 mL and 10 mL respectively.
$$\left[\log(25.6) = 1.408; \ \log(2) = 0.301\right]$$

28. Order of reaction is _____ .

29. Time in (min) for 50% completion of the reaction will be _____ .

SECTION – 3 (Maximum Marks : 24)

This section contains **SIX (06)** questions.
Each question has FOUR options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
For each question, choose the correct option(s) to answer the question.
Answer to each question will be evaluated according to the following marking scheme:
Full Marks :   +4 If only (all) the correct option(s) is (are) chosen.
Partial Marks: +3 If all the four options are correct but ONLY three options are chosen.
Partial Marks: +2 If three or more options are correct but ONLY two options are chosen, both of which are correct options.
Partial Marks :   +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.
Zero Marks :   0 If none of the options is chosen (i.e. the question is unanswered).
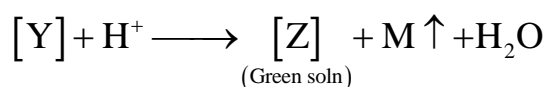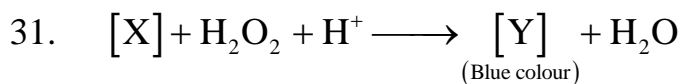**Negative Marks:** -2 In all other cases.

30. Which of the following statement(s) is/are correct?

A) Cationic part of Cl_2O_6 is planar

B) $N(SiH_3)_3$ is more basic than $N(CH_3)_3$

C) HNO_2 and $HClO_3$ both disproportionate on heating

D) NO_2 and ClO_2 both are sp^2 hybridized

31. $$[X] + H_2O_2 + H^+ \longrightarrow \underset{\text{(Blue colour)}}{[Y]} + H_2O$$

$$[Y] + H^+ \longrightarrow \underset{\text{(Green soln)}}{[Z]} + M\uparrow + H_2O$$

Now, identify the correct statement(s) regarding above sequence of reactions.

A) 'X' may be K_2CrO_4 or $K_2Cr_2O_7$

B) 'Y' is CrO_3

C) Green colour of (Z) is due to the presence of Cr^{3+}

D) Gas (M) is O_2

32. Pick out the **correct** statements among the following from inspection of standard reduction potentials (Assume standard state conditions).

$Cl_2(aq.) + 2e^- \rightleftharpoons 2Cl^-(aq.) = +1.36$ volt

$Br_2(aq.) + 2e^- \rightleftharpoons 2Br^-(aq.) = +1.09$ volt

$I_2(s) + 2e^- \rightleftharpoons 2I^-(aq.) = +0.54$ volt

$S_2O_8^{2-}(aq.) + 2e^- \rightleftharpoons 2SO_4^{2-}(aq.) = +2.00$ volt
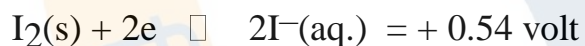
A) Cl_2 can oxidise SO_4^{2-} from solution

B) Cl_2 can oxidise Br^- and I^- from aqueous solution

C) $S_2O_8^{2-}$ can oxidise Cl^-, Br^- and I^- from aqueous solution

D) $S_2O_8^{2-}$ can reduce Br^-

33. Among the following " TRUE " statements are :

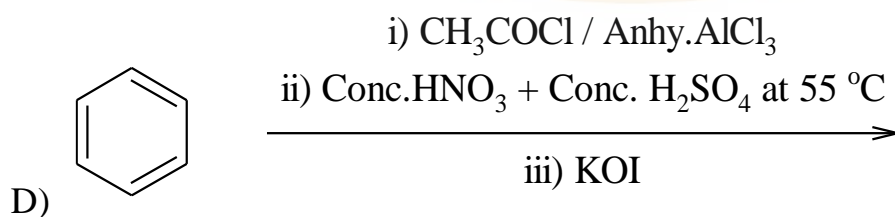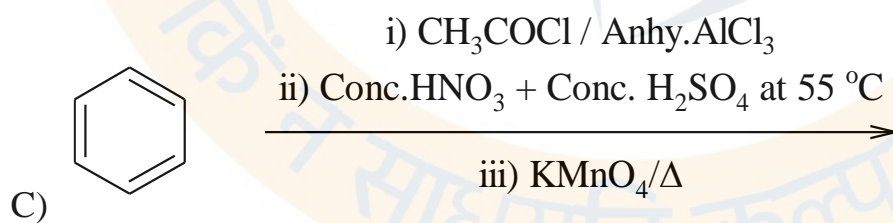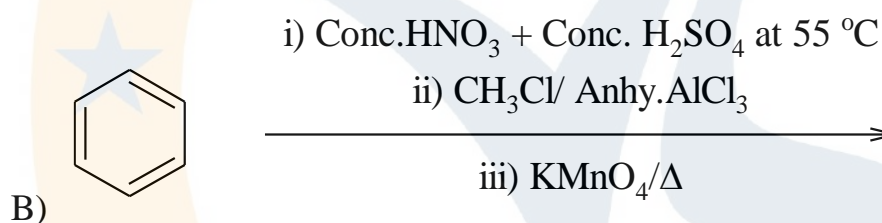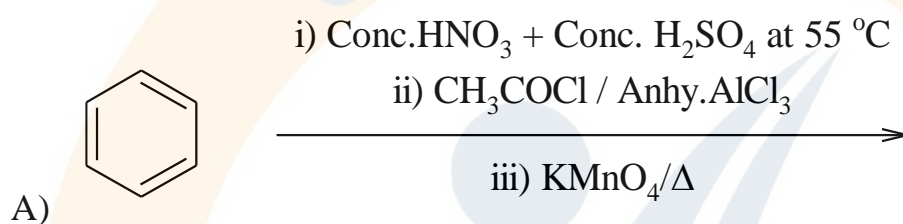(A) Reversible adiabatic processes are ISOENTROPIC in nature.

(B) $\left(\Delta S_{sys}\right)$ for reversible adiabatic expansion is zero since the increase in entropy

caused by the expansion compensates the decrease in entropy due to cooling.

(C) $\left(\Delta S_{surr}\right)$ for adiabatic reversible and irreversible processes will be zero.

(D) For an isothermal free expansion, $\Delta U = 0; \Delta H = 0; w = 0; q = 0$

34. Correct reaction(s) to prepare metanitro benzoic acid/salt is (are)

A)
$$\xrightarrow[\text{iii) KMnO}_4/\Delta]{\substack{\text{i) Conc.HNO}_3 + \text{Conc. H}_2\text{SO}_4 \text{ at 55 }^o\text{C} \\ \text{ii) CH}_3\text{COCl / Anhy.AlCl}_3}}$$

B)
$$\xrightarrow[\text{iii) KMnO}_4/\Delta]{\substack{\text{i) Conc.HNO}_3 + \text{Conc. H}_2\text{SO}_4 \text{ at 55 }^o\text{C} \\ \text{ii) CH}_3\text{Cl/ Anhy.AlCl}_3}}$$

C)
$$\xrightarrow[\text{iii) KMnO}_4/\Delta]{\substack{\text{i) CH}_3\text{COCl / Anhy.AlCl}_3 \\ \text{ii) Conc.HNO}_3 + \text{Conc. H}_2\text{SO}_4 \text{ at 55 }^o\text{C}}}$$

D)
$$\xrightarrow[\text{iii) KOI}]{\substack{\text{i) CH}_3\text{COCl / Anhy.AlCl}_3 \\ \text{ii) Conc.HNO}_3 + \text{Conc. H}_2\text{SO}_4 \text{ at 55 }^o\text{C}}}$$

35. Correct statement(s) about the heating of the given compound is (are)
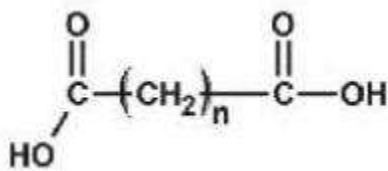
When n=1, organic product formed is P_1

When n=3, organic product formed is P_2

When n=4, organic product formed is P_3

A) P_1 gives yellow solid with NaOI

B) P_2 is soluble in hot aq. NaOH solution

C) P_3 gives colored solid with Brady's reagent

D) P_1 is more acidic than P_3

SECTION - 4 (Maximum Marks : 12)

This section contains **THREE (03)** questions. The answer to each question is a **NON-NEGATIVE INTEGER**
For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter answer.
Answer to each question will be evaluated <u>according to the following marking scheme:</u>
Full Marks: +4 If ONLY the correct numerical value is entered as answer.
Zero Marks: 0 In all other cases.

36. How many of the following reactions evolve a paramagnetic gas?

(a) $Cu + dil.HNO_3 \rightarrow$

(b) $Zn + dil\,HNO_3 \rightarrow$

(c) $Pb + dil\,HNO_3 \rightarrow$

(d) $Zn + NaOH \rightarrow$

(e) $(NH_4)_2\,SO_4 + NaOH \rightarrow$

(f) $KO_2 + H_2O \rightarrow$
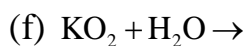
37. The number of square faces and regular hexagons in a truncated octahedron are collectively found to be " a " and rank of a unit cell of SiC (identical to diamond crystal lattice) is found to be " b ". Value of (a – b) will be

38. The absolute value of the net charge of the given peptide at $pH = 7$ is-----

Hint: The following table gives the pKa values for the α-carboxylic acid group, the α-amino group, and any ionizable side chains for different α-amino acids

Amino Acid	α-carboxylic acid	α-amino	Side chain
Cysteine	2.05	10.25	8.0
Glycine	2.35	9.78	-
Histidine	1.77	9.18	6.10
Lysine	2.18	8.95	10.53
Tyrosine	2.20	9.11	10.07

SECTION – 1 (Maximum Marks: 12)

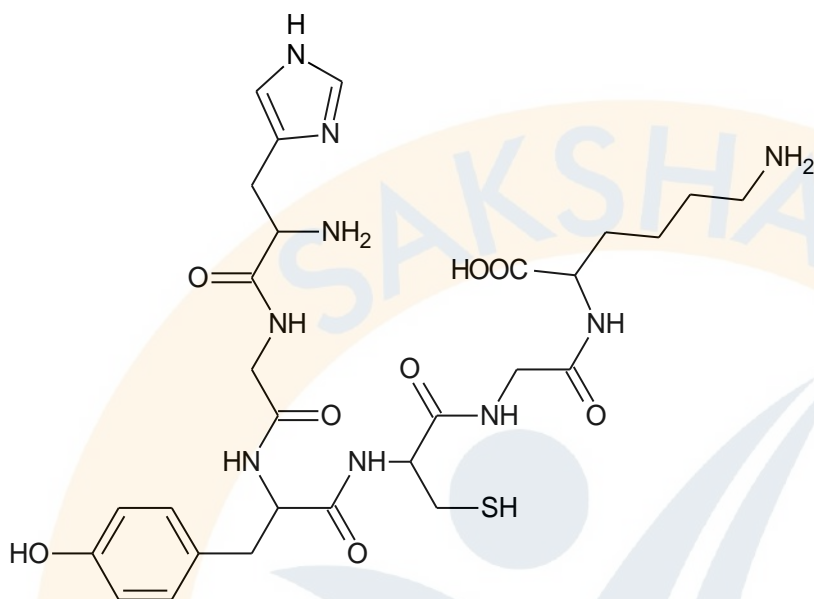This section contains **SIX (04)** questions.

Each question has FOUR options for correct answer(s). **ONLY ONE** of these four option is the correct answer.

For each question, choose the correct option corresponding to the correct answer.

Answer to each question will be evaluated <u>according to the following marking scheme</u>:

Full Marks : +3 If only the correct option is chosen.

Zero Marks: 0 If none of the option is chosen.(i.e the question is un answered)

Negative Marks: -1 In all other cases.

39. Let R be the region in the first quadrant bounded by the x-axis, the line $2y = x$, and the ellipse $\dfrac{x^2}{9} + y^2 = 1$. Let R' be the region in the first quadrant bounded by the y-axis, the line $y = mx$, and the ellipse $\dfrac{x^2}{9} + y^2 = 1$, then the value of m such that R and R' have the same area is

   A) $\dfrac{2}{9}$ B) $\dfrac{3}{11}$ C) $\dfrac{2}{5}$ D) $\dfrac{2}{7}$

40. Suppose $P(x)$ is a cubic polynomial with integer coefficients such that $P(\sqrt{5}) = 5$ and $P(\sqrt[3]{5}) = 5\sqrt[3]{5}$ then $P(5)$ is equal to

   A) 95   B) –95   C) 75   D) –75

41. For all $n \geq 2$,let $a_n = \displaystyle\sum_{k=1}^{n-1} \dfrac{\sin\left(\dfrac{(2k-1)\pi}{2n}\right)}{\cos^2\left(\dfrac{(k-1)\pi}{2n}\right)\cos^2\left(\dfrac{k\pi}{2n}\right)}$ and $L = \displaystyle\lim_{n\to\infty} \dfrac{a_n}{n^3}$ then $\left[\dfrac{1}{L}\right]$ is equal to (where [.] denotes greatest integer function)

   A) 2   B) 3   C) 4   D) 1

42. Let $f(x)$ be a differentiable function such that $f^{2020}(0)$ exists ($f^n(x) = n^{th}$ order derivative of $f(x)$ with respect to x). If there exists an even function $g(x)$ that satisfies $g(x) = \left(\dfrac{(2019)^x - (2019)^{-x}}{(2019)^x + (2019)^{-x}}\right) f(x), \forall x \in \mathbf{R}$, then $f^{2020}(0)$ is equal to –

   A) 1   B) – 1   C) 0   D) 2020

SECTION - 2 (Maximum Marks : 12)

This section contains **TWO (3)** Paragraphs. Based on each paragraph, there are **2** questions.
The answer to each question is a **NUMERICAL VALUE**.
For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter answer. If the numerical value has more than two decimal places **truncate/round- off** the value to **TWO** decimal places.
Answer to each question will be evaluated <u>according to the following marking scheme:</u>
Full Marks: +2 If ONLY the correct numerical value is entered as answer.
Zero Marks: 0 In all other cases.

Question Stem for Question Nos. 43 and 44:

Consider the parallelogram with vertices $(10,45),(10,114),(28,153),$ and $(28,84)$. A line through

the origin cuts this figure into two congruent polygons. The slope of the line is $\dfrac{m}{n}$,

where m and n are relatively prime positive integers, then

43.   m is equal to_____

44.   n is equal to_____

Question Stem for Question Nos. 45 and 46:

A parabola touches the bisectors of the angle obtained by the lines $x+2y+3=0$ and

$2x+y+3=0$ at the point $M(1,1)$ and $N(0,-2)$.

45.   The absolute value of the sum of the x and y coordinates of the focus of the parabola is

46.   If the equation of the directrix is given as $ax+by+4=0$ then the value of $a+b$ is

Question Stem for Question Nos. 47 and 48:

Consider the locus of the complex number Z in the Argand plane given by $\mathrm{Re}(z)-2=|z-7+2i|$.

Let $P(z_1),Q(z_2)$ be two complex numbers satisfying the given locus and also satisfying

$\arg\left(\dfrac{z_1-(2+\alpha i)}{z_2-(2+\alpha i)}\right)=\dfrac{\pi}{2}(\alpha\in R)$ (Where $\arg z\in(-\pi,\pi]$)

47.   The minimum value of PR.QR where R represents the point $(7,-2)$ is

48. $\arg\left(\dfrac{z_1-(7-2i)}{z_2-(7-2i)}\right)$ equals to

SECTION - 3 (Maximum Marks : 24)
This section contains **SIX (06)** questions.
Each question has FOUR options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
For each question, choose the correct option(s) to answer the question.
Answer to each question will be evaluated according to the following marking scheme:
Full Marks :   +4 If only (all) the correct option(s) is (are) chosen.
Partial Marks: +3 If all the four options are correct but ONLY three options are chosen.
Partial Marks: +2 If three or more options are correct but ONLY two options are chosen, both of which are correct options.
Partial Marks :   +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.
Zero Marks :   0 If none of the options is chosen (i.e. the question is unanswered).
Negative Marks: -2 In all other cases.

49. A line touches a hyperbola at P and intersects the pair of asymptotes at points Q and R. If equation of pair of asymptotes is given by $4x^2 + 8xy - y^2 = 0$ and mid-point of QR is $(1,0)$, then

A) equation of hyperbola is given by $4x^2 + 8xy - y^2 - 4 = 0$

B) length of transverse axis of hyperbola is $\dfrac{\sqrt{89} - 3}{10}$

C) If circumcenter of $\triangle OQR$ is (a,b) then $a + 9b = 4$; (O is center of hyperbola)

D) If circumcenter of $\triangle OQR$ is (a,b) then $a - b = -1$; (O is center of hyperbola)

50. Let $\sin^{-1} x : [-1,1] \to \left[\dfrac{\pi}{2}, \dfrac{3\pi}{2}\right]$ and $\cos^{-1} x : [-1,1] \to [\pi, 2\pi]$. If $\dfrac{\left(\sin^{-1} x\right)^2}{3\pi} + \dfrac{\left(\cos^{-1} x\right)^2}{4\pi} = \dfrac{25}{28}\pi$, then

A) $\sin^{-1} x \in \left[\pi, \dfrac{5\pi}{4}\right]$

B) $\sin^{-1} x \in \left[\dfrac{5\pi}{4}, \dfrac{3\pi}{2}\right]$

C) $\cos^{-1} x \in \left[\pi, \dfrac{5\pi}{4}\right]$

D) $\cos^{-1} x \in \left[\dfrac{5\pi}{4}, \dfrac{3\pi}{2}\right]$

51. Let a_n and b_n be the sequences of real numbers such that $(2+i)^n = a_n + b_n i$ for all integers $n \geq 0$ where $i = \sqrt{-1}$, then

A) $\sum\limits_{n=0}^{\infty} \dfrac{a_n b_n}{7^n} = \dfrac{7}{16}$

B) $\sum\limits_{n=0}^{\infty} \dfrac{a_n b_n}{7^n} = \dfrac{1}{20}$

B) $\sum\limits_{n=0}^{\infty} \dfrac{a_n^2 - b_n^2}{7^n} = -\dfrac{1}{20}$

D) $\sum\limits_{n=0}^{\infty} \dfrac{a_n^2 - b_n^2}{7^n} = \dfrac{7}{8}$

52. If $f(x) = \big\|\,|x-1| + |x-3| - |2x-1|\,\big\| + \big\|\,|x-1| + |x-3| + |2x-1|\,\big\|$, then

A) $f(x)$ is non-differentiable at $x=1$

B) $f(x)$ is non-differentiable at $x=\dfrac{3}{2}$

C) $f(x)$ has minimum value is 4

D) f(x) is non-differentiable at $x=\dfrac{1}{2}$

53. Suppose $I_1 = \displaystyle\int_0^{\frac{\pi}{2}} \cos\left(\pi \sin^2 x\right)dx$, $I_2 = \displaystyle\int_0^{\frac{\pi}{2}} \cos\left(2\pi \sin^2 x\right)dx$, $I_3 = \displaystyle\int_0^{\frac{\pi}{2}} \cos\left(\pi \sin x\right)dx$ then

A) $I_1 = 0$ B) $I_2 + I_3 = 0$ C) $I_1 + I_2 + I_3 = 0$ D) $I_2 = I_3$

54. Let $S = \{1,2,3,4,.....,n\}$ and f_n be the number of those subsets of S which do not contain consecutive elements of S, then (empty set is taken as subset of S which satisfying given condition)

A) $f_n = \dfrac{n(n-1)(n-2)}{6}$

B) $f_n = 2f_{n-1}$

C) $f_n = f_{n-1} + f_{n-2}$

D) $f_4 = 8$

SECTION - 4 (Maximum Marks : 12)

This section contains **THREE (03)** questions. The answer to each question is a **NON-NEGATIVE INTEGER**
For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter answer.
Answer to each question will be evaluated <u>according to the following marking scheme:</u>
Full Marks: +4 If ONLY the correct numerical value is entered as answer.
Zero Marks: 0 In all other cases.

55. If a and b are positive real numbers such that $a(2^b) = 8$ and $(a)^b = 2$ then the value of
$$\dfrac{(a^{\log_2 a})(2^{b^2})}{64} =$$

56. Two lines are formed by intersection of plane $2x + 3y + 4z - 1 = 0$ with the planes $x + y + z - 3 = 0$ and $x + y + z + 3 = 0$, if the shortest distance between both the lines is $\sqrt{(29)(6^k)}$ then k is equal to_____

57. Let $\vec{b} = 4i + 3j$ and $\vec{c}$ be two vectors perpendicular to each other in the xy-plane. If $\vec{r_i}$, $i = 1,2,...n$ are distinct vectors in the same plane having length of projections 1 and 2 along $\vec{b}$ and $\vec{c}$ respectively then $\dfrac{\displaystyle\sum_{i=1}^{n}|\vec{r_i}|^2}{10}$ is equal to_____